BOUNDARY SHEAR DISTRIBUTION IN MEANDERING CHANNEL

A Thesis Submitted in Partial Fulfilment of the Requirement for the Degree Of

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In

Civil Engineering

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DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

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Submitted by

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In partial fulfilment of the requirements

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Under the Supervision of

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DECLARATION

I hereby state that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by any other person nor substance which to a substantial extent has been accepted for the award of any other degree or diploma of the university or other institute of higher learning, except where due acknowledgement has been made in the text.

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CERTIFICATE

This is to certify that the thesis entitled **"Boundary Shear Distribution in Meandering Channel"** is a bonafide record of authentic work carried out by **Aishwarya Nayak** under my supervision and guidance for the partial fulfilment of the requirement for the award of **Master of Technology** degree in **Civil Engineering** with specialization in **Water Resources Engineering** at the National Institute of Technology, Rourkela.

The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Date:

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Aishwarya Nayak

ABSTRACT

For many years, river modelling has been a core subject in the field of hydraulics. Over the past few decades, various attempts have been made to build models for flow. The distribution of shear stress over the wetted perimeter of a meandering open channel is one of the common river engineering problem. It is a key parameter to determine bank erosion and river migration. The influence of turbulence, the secondary flow, the curvature effect, the shape of the cross-section and the boundary condition are some among many technical hitches that are generally related in one way or another to the boundary shear stress making it difficult to analyse. Although some recent works have shed light on the phenomenon of boundary shear distribution, there are still several unexplored aspects that worth consideration.

The research work which is presented in this thesis is an attempt to devise an analytical method, which could compute the average shear stress acting on the bed, inner wall and outer wall of a meandering channel. The attempt relies on splitting the channel cross-section into sub-regions and computing respective hydraulic radii by energy balance method. The concept of energy transportation, well studied, by *Einstein* (*1942*) and *Yang and Lim (1997)* is to be used for first achieving a theoretical basis and then modifying it with perceptive changes to account the curvature of a meandering channel.

Keywords: *Meander, Boundary Shear, Energy Balance, RANS, Order of Magnitude, Flow Division*

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SYMBOL	DESCRIPTION
A	Cross-sectional Area of Channel
b	Channel Width
C	Chezy's channel coefficient
C_d	Coefficient of Discharge
g	Acceleration due to Gravity
h	Flow Depth
h_{cr}	Depth of Water At Critical Condition
H	Average flow Depth of water at a Section
h_w	Height of Water
H_n	Height of water above the Notch
L	Length of Channel for one Wavelength
L_n	Length of Rectangular Notch
n	Manning's Roughness Coefficient
ΔP	Differential Pressure
Q_a	Actual Discharge
Q_{th}	Theoretical Discharge
r_c	Radius of Curvature of a Sinuous Channel
ρ	Density of the Flow
S	Bed Slope of the Channel
S_v	Slope of Valley
S_r	Sinuosity
S_f	Slope of Energy Line
SF_{Bed}	Shear Force at the Bed of the Channel
SF_{Inner}	Shear Force at the Inner Wall of the Channel Section
SF_{Outer}	Shear Force at the Outer Wall of the Channel Section

SF_T	Total Shear Force
τ_c'	Average Shear Stress
τ	Boundary Shear Stress
υ	Kinematic Viscosity
V_w	Volume of Water
v	Point Velocity
W	Width of Channel
x^*, y^*	Non-Dimensional Parameters
λ	Wavelength of a Sinuous Channel
α_{cr}	Critical Aspect Ratio
C	Centrifugal Ratio
Φ	Relative Distance
Ψ	Geometric Distance
k	Roughness of Height
μ_*	Friction Velocity
υ	Kinematic Velocity
r_c	Radius of Curvature of Channel
L_c	Channel Length
L_v	Straight Line Valley Length
D_s	Thickness of Viscous Sub-layer
V_M	Mean Flow Velocity

CHAPTER I

INTRODUCTION

1.1. General

"Rivers know this: there is no hurry, we shall get there someday – A.A. Milne".

For many years, river modelling has been a core subject in the field of hydraulics. Over the past few decades, various attempts have been made to build models for flow. The distribution of shear stress over the wetted perimeter of a meandering open channel is one of the common river engineering problem. It is a key parameter to determine bank erosion and river migration. The influence of turbulence, the secondary flow, the curvature effect, the shape of the cross-section and the boundary condition are some among many technical hitches that are generally related in one way or another to the boundary shear stress making it difficult to analyse. Although some recent works have shed light on the phenomenon of boundary shear distribution, there are still several unexplored aspects that worth consideration.

Accurate computation of the local or mean shear stress is a difficult task even using sophisticated turbulence models. As an alternative, various empirical, analytical or simplified computational methods were developed to calculate the open channel boundary shear stresses. Leighly (1932), Einstein (1942), Ghosh and Roy (1970), Kartha and Leutheusser (1970), Knight and Macdonald (1979), Knight (1981), Knight et al. (1984), Knight and Patel (1985), Knight and Sterling (2000), Yang and McCorquodale (2004), Lashkar and Fathi (2010) carried out ample studies in straight channels on boundary shear computation. Research work has also been carried out on meandering channel for predicting the boundary shear. Most of them are through large experimentation and then determining empirical model using the experimental evidences. Knight, Yuan and Fares (1992), Shiono, Muto and Knight (1999), Ervine, Alan, Koopaei and Sellin (2000), Patra and kar (2000), Khatua (2008) and many research works introduced and examined various aspects of meandering channel. Regardless of this vast work carried out, limited information exists in literature to describe the actual fraction from the flow field required to calculate the sidewall and bed shear stress distributions along the channel cross section of a meandering channel. Yang and Lim (1997) and Guo and Julien (2005) has proposed two such mechanism to divide the flow field, but both of them are on straight channels which gives equal distribution of shear on both the banks. Hence this can cannot be applied on channel with extensive curvature where the shear distribution is unequal on both the banks.

1.2. Problem Statement

In a meandering channel flow, the boundary shear stress and its distribution along the boundary is of great importance. A knowledge of this variable, be it local or mean values, is required in many hydraulic problems associated with computation of flow resistance, side wall correction, sediment transport rate, channel erosion or deposition and designs of the channels for the long term stability (Yang & Lim, 1998) and most pertinently channel migration that occurs in a meander river with time.

The problem of separating the bed shear stress, inner and outer wall shear stress from the total shear stress acting on a channel is of great importance. Experimental evidences are also available in the literature which shows, there is non-uniform distribution of shear stress on both the banks of a meander channel. Higher at the inner bank and lower at the outer bank. All this three stress has individual significance. Say, for bed sediment transport bed shear stress is required and for study of channel migration one must know the shear stress acting at the concave side of a curve to predict the amount of erosion it might lead to. This can be achieved either by rigorous measurement or by certain theoretical analysis to devise an analytical model, which will compute the same.

The research work which is presented in this thesis is an attempt to devise an analytical method, which could compute the average shear stress acting on the bed, inner wall and outer wall of a meandering channel. The attempt relies on splitting the channel cross-section into sub-regions and computing respective hydraulic radii by energy balance method. The concept of energy transportation, well studied, by *Einstein* (*1942*) and *Yang and Lim (1997)* is to be used for first achieving a theoretical basis and then modifying it with perceptive changes to account the curvature of a meandering channel.

1.3. Objectives

The main aim of the work is to divide the flow cross section into three sub-regions to compute average shear stress on the bed and walls of a meandering channel. To attain this the following objectives are defined –

(i) Generalize a method to divide the main flow cross section into its various subsections inside which energy will be balanced, the weight of fluid is balanced by shear force acting along the corresponding wall sections for computation of the mean wall and the mean bed shear stress in meandering channels.

(ii) Study of steady non-uniformity characteristics of flow and introducing the same into the model for correct prediction.

(iii) Validation of the proposed model with other available experimental data.

1.4. Organization of the thesis

The thesis consists of five chapters, General introduction and the problem statement is provided in Chapter 1, Chapter 2 contains basic concepts of a channel, literature reviews are given in Chapter 3, Chapter 4 covers the details on model derivation, Chapter 5 deals with evaluation of the model and finally Chapter 6 gives the conclusions drawn from the analysis and then the references are presented.

Chapter 2 focuses on the basic concepts of meanders, channel conditions and flow mechanisms

In Chapter 3, a literature review is presented on various works carried out by different researchers.

Chapter 4 covers the theoretical considerations and analytical formulations done to devise the model to predict the average shear stress on bed, inner and outer bank of a meander channel at the bend apex. It also details the basic assumptions and limitation of the model.

Chapter 5 focuses on the result generated from the model. Various experimental measurements carried out by different research work are used to compare with model generated output for model validation.

In Chapter 6, the conclusions of the research are presented.

CHAPTER II

BASIC CONCEPTS

2.1 Overview

This chapter provides a brief description of the key concepts, mechanisms and other characteristics of a meandering channel. The term fluvial is commonly used to describe the processes associated with rivers or streams, and the erosion or deposition and morphology created by them. The fluvial processes include transport of sediments and aggregations or degradation of river beds. The flow in a bed formed by the loose sediment exerts shear stress on the bed. The fluid flows as a result of the action of the forces introduced by the difference of pressure or gravity. The movement of the fluid is controlled by the fluid inertia and the effect of the shear stress exerted by the surrounding fluid. As the fluid flows in a solid boundary, either stationary or in motion, the velocity of the fluid in contact with the boundary must be the same as the boundary, called no slip. Therefore, a velocity gradient is created at the boundary, because the velocity of the fluid increases with the normal distance from the boundary. The resulting differential normal velocity to the boundary give rise to shear stress in the fluid and on the boundary, as suggested by Newton's law of viscosity. (Dey, 2004)

2.2 Meander

Leopold and Langbein (1966), Meanders are distinct curves or forms in which a river does the least work in turning. A meander, in general, is a bend in a sinuous watercourse or river. It forms in a river when material is eroded from the concave portion, transported towards downstream and deposited on the convex portion of a meander.

Meander path is a flow path undertaken by a river. Bend apex or the axis of bend in the section at which the river has the maximum curvature. A channel while moving from one bend apex to the other passes through the cross-over. Cross-over is a section at the point of inflection where the meander path changes its course as shown in Fig. 2.1. *'w'* represents the width of the channel, λ represents the wavelength, L represents the length of channel for one wavelength and r_c represents the radius of curvature of the channel.

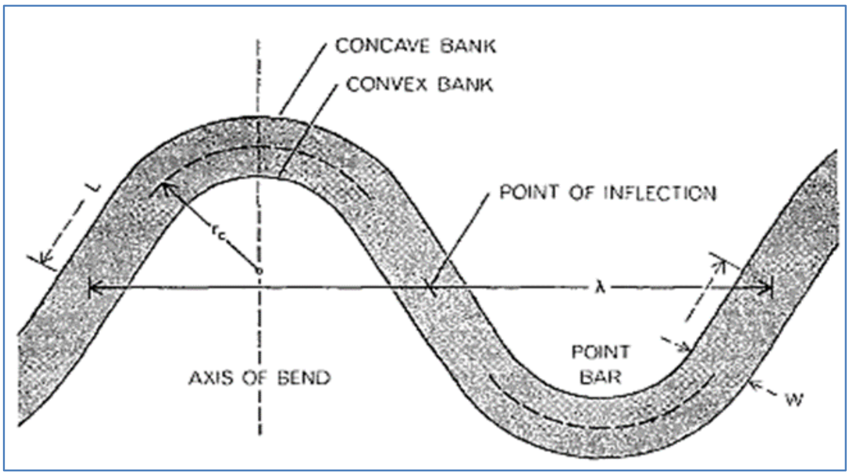


Fig. [2.1]: Properties of River Meander (Leopold and Langbein, 1966)

2.3 Sinuosity

Sinuosity is used to describe meandering channel geometry. When sinuosity is greater than 1.5, the channel is classified as meandering, *Knighton (1998)*. Equation [2.1] used for calculating channel sinuosity-

$$\text{Sinuosity, } S_r = \frac{L_c}{L_v} \qquad [2.1]$$

Where L_c= the channel length

L_v= the straight line valley length.

2.4 Flow Through Meandering Channel

When a flow enters a meander it is influenced by radial force due to centrifugal action. This induces a three dimensionality in the flow characterized by a helical (spiral) motion with a super-elevated free surface. The helical motion can be viewed across a cross section as a transverse circulation. The differential centrifugal acceleration u^2/r along a vertical line due to vertical variation of stream wise velocity u (greater near surface and less near bed) in open channel is the cause of the transverse circulation. As a result, a helical motion is initiated when the flow enters the curved (bend) portion of the channel. The streamlines near the free surface are deflected toward the outer bank, whereas those near the bed are inclined toward the inner bank. Hence, the near-bed velocity and the bed shear stress are generally directed toward the inner bank.

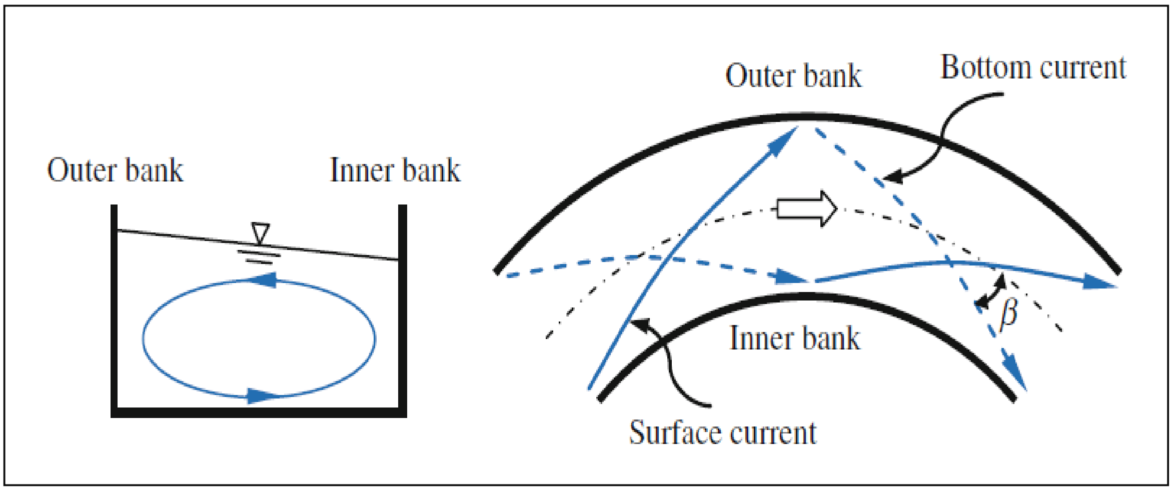


Fig [2.2]- Flow in a curved section of meandering channel (Subhashish Dey, Fluid Hydrodynamics, 2014)

2.5 Shear Velocity

Shear velocity or friction velocity is a form by which a shear stress is written in units of velocity. According to *Chow (1959)* it can be defined as:

$$u_* = \sqrt{\frac{\tau_0}{\rho}} \qquad [2.2]$$

In which τ_0 is the bed-shear stress (bottom shear stress) and ρ is the density of fluid.

2.6 Channel Roughness

Roughness or hydraulic roughness is the measure of the amount of frictional resistance water experiences when passing over the channel features. The effective height of the irregularities forming the roughness elements is called the roughness height, *Chow (1959).* It is denoted by k.

The flow can be hydraulically either smooth or rough. Hydraulically smooth flow occurs when the surface irregularities are so small that all roughness elements are entirely submerged in the laminar sub-layer, *Chow (1959).* Therefore, the bed roughness will not affect the velocity distribution. According to *Graf (1998)* and *Schlichting & Gersten (2000)* the flow is smooth if

$$0 < \frac{u_* k}{\nu} < 5 \qquad [2.3a]$$

where u* is the friction velocity [m/s]; and k is the roughness height [mm]. The flow is rough when bed roughness is so large that it produces eddies close to the bottom, *Liu (2001).* There is no viscous sub-layer and the velocity distribution is affected only by bed roughness. According to *Graf (1998)* and *Schlichting & Gersten (2000)* the flow is hydraulically rough if

$$70 < \frac{u_* k}{\nu} \qquad [2.3b]$$

For the term hydraulically rough, also term fully rough is used *(Schlichting & Gersten, 2000)*. The flow is in the transition region if

$$5 < \frac{u_* k}{\nu} < 70 \qquad [2.3c]$$

then the velocity distribution is affected by bed roughness and viscosity *(Chow, 1959).*

2.7 Law of Wall

The law of the wall states that the average velocity of a turbulent flow at a certain point is proportional to the logarithm of the distance from that point to the wall, or the boundary of the fluid region as first published by *Theodore von Karman (1930)*. It is only technically applicable to parts of the flow that are close to the wall, though it is a good approximation for the entire velocity profile of natural streams. The logarithmic law of wall can be expressed, *Schlichting (2000)*, as:

$$u = u_* A \ln\left(\frac{u_* y}{\nu}\right) + B \qquad [2.4a]$$

Where u is the velocity parallel at a distance y from the wall,

u_* is the shear velocity

ν is the kinematic viscosity

A and B are the constants. Where A is reciprocal of von Karman constant

This inner layer comprises of viscous sub-layer, buffer layer and log-law region.

For two-dimensional flows the shear stress in the viscous sub-layer is considered to be constant, and is given by:

$$\tau_o = \mu\left(\frac{\partial u}{\partial y}\right) \qquad [2.4b]$$

which after integration yields:

$$\frac{u}{u_*} = \frac{u_* y}{\nu} \qquad [2.4c]$$

The viscous sub-layer is extremely thin and as a consequence equation [2.4c] is only valid for the range $(u_* y)/\nu < 5$. Outside this layer both viscous and turbulent effects are important. For $5<(u_* y)/\nu<30$, the buffer layer, neither equation [2.4a] nor [2.4c] holds.

2.8 Boundary Shear

Resistance from bed and side slope is exerted on the water flowing in an open chann el.This resistance force is apparently the boundary shear force. Boundary shear stress is the tangential component of the hydrodynamic forces acting along the channel bed. Flow characteristics of an open channel flow are directly dependent on the boundary shear force distribution along the wetted perimeter of the channel.

The shear force, for steady uniform flow is related to bed slope, hydraulic radius and unit weight of fluid. However, in a practical point of view, these forces are not unifo rm even for straight prismatic channels. The nonuniformity of shear stress is mainly d ue to secondary currents formed by the anisotropy between vertical and transverse tur bulent intensities, given by Gessner (1973). Boundary shear stress increases when the secondary currents flow towards the wall and shear stress decreases when it flows aw ay from the wall, Tominagaet al. (1989) and Knight and Demetriou (1983). The prese nce of secondary flow cells in main channel affects the distribution of shear stress al ong the channel's wetted perimeter which is illustrated in Fig. 2.3. Other factors affec ting the shear stress distribution are the shape of channel crosssection, depth of flow, later longitudinal distribution of wall roughness and sediment concentration. For the case of meandering channels, the factors increase even more due to the nature of flo w of water in such channels. Sinuosity in the case of meandering channel is regarded to be a critical parameter in the shear stress distribution along the channel bed and walls

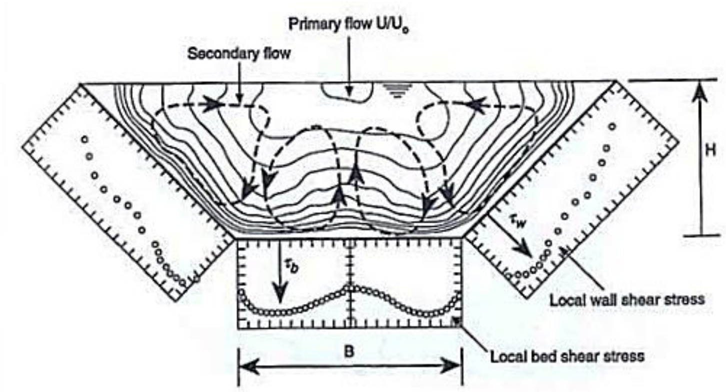


Fig.2.3: Schematic influence of secondary flow cells on boundary shear distribution in a trapezoidal section (Knight *et al.*, 1944)

CHAPTER III

LITERATURE REVIEWS

3.1 Overview

In this chapter a detailed literature survey of various research works are given which covers various aspects concerning the meandering channels and boundary shear.

3.2 Reviews on past research works

Leighly (1932) - idea to use *conformal mapping* was first suggested by him. By neglecting the effects of secondary currents, the boundary shear stress acting on the channel bed must be balanced by the component of the weight contained between each of the two adjacent orthogonals, but his proposal did not render any conclusive results.

Einstein (1942) - divided the channel flow cross section into two subsections that correspond to the channel bed and sidewalls, this division is made by an imaginary separation boundary, namely the *division curve*. Although Einstein did not propose any method for determining the exact location of the division curves, his suggestion laid the foundation for the development of methods used to isolate the flow cross section into bed and sidewall sub-sections.

Rozovskii (1961) - analysed the flow characteristics in a meandering channel and proposed a method to compute the *radial shear stress* in a meandering channel. He even provided a comparison and discussion of other researcher s' studies.

Ghosh and Roy (1970)-
presented the boundary shear distribution in both rough and smooth open channels of rectangular and trapezoidal sections obtained by direct measurement of shear drag on an isolated length of the test channel utilizing the technique of three point suspension system suggested by Bagnold. Existing shear measurement techniques were reviewed critically.Comparisons were made of the measured distribution with other indirect esti mates, from isovels, and Preston tube measurements. The discrepancies between the direct and indirectestimates were explained and out of the two indirect estimates the Pitot tube technique was found to be more reliable. The influence of secondary flow on the boundary shear distribution was not accurately defined in the absence of a de pendable theory on secondary flow.

Kartha and Leutheusser (1970)- expressed that the designs of alluvial channels by the tractive force method requires information on the distribution of wall shear stress over the wetted perimeter of the cross-section. The experiments were carried out in a smooth-walled laboratory flume at various aspect ratios of the rectangular cross-section. Wall shear stress measured with

Preston tubes were calibrated by a method exploiting the logarithmic form of the inner law of velocity distribution. Results were presented which clearly suggested that none of the present analytical techniques could be counted upon to provide any precise details on tractive force distribution in turbulent channel flow.

Ghosh and Kar (1975) - studied the evaluation of interaction effect and the distribution of boundary shear stress in meander channel with floodplain. Using the relationship proposed by Toebes and Sooky (1967) they evaluated the interaction effect by a parameter (W). They concluded that the channel geometry and roughness distribution did not have any influence on the interaction loss.

Knight (1981) -proposed an empirically derived equation that presented the percentage of the shear force carried by the walls as a function of the breadth/depth ratio and the ratio between the Nikuradse equivalent roughness sizes for the bed and the walls. The results were compared with other available data for the smooth channel case and some disagreements noted. The systematic reduction in the shear force carried by the walls with increasing breadth/depth ratio and bed roughness was illustrated. Further equations were presented giving the mean wall and bed shear stress variation with aspect ratio and roughness parameters. Although the experimental data was somewhat limited, the equations were novel and indicated the general behaviour of open channel flows with success.

Knight and Patel (1985)- reported some of the experimental results concerning the distribution of boundary shear stresses in smooth closed pipes of a rectangular cross section for aspect ratios between 1to10.The distributions were shown to be influenced by the number and shape of the secondary flow cells, which, in turn, depended primarily upon the aspect ratio.For a square cross section with 8 symmetrically disposed secondaryflow cells,a double peak in the distribution of the boundary shear s tress along each wallwas shown to displace the maximum shear stress away from the centre position towards each corner. For rectangular cross sections, the number of se condary flow cells increasedfrom 8 by increments of 4 as the aspect ratio increased, causing alternate perturbations in the boundary shear stress distributions at positions w here there were adjacent contra-rotating flow cells. Equations were presented for the maximum, centreline and mean boundary shear stresses on the duct walls in terms of the aspect ratio.

Chein and Wan (1986) – They concisely explained the physical meaning of Einstein's idea in terms of how the surplus energy in main flow should be transferred and eventually dissipated as heat at the boundary. According to them, it is physically impossible for energy contained in any unit flow volume to be transported in all direction towards the boundary and hence it should transport towards the nearest boundary with constant energy slope.

Knight, Yuan and Fares (1992) - reported the experimental data of SERC-FCF concerning boundary shear stress distributions in meandering channels throughout the path of one complete wave length. They examined the effects of secondary currents, channel sinuosity, and cross section geometry on the value of boundary shear in meandering channels and presented a momentum-force balance for the flow.

Yang and Lim (1997, 1998) - gave an *analytical approach* for partitioning the flow cross-section for steady and uniform three-dimensional channels. Their approach is based on the premise that the surplus energy of any arbitrary unit volume of fluid in three dimensional flow channels will be transmitted towards and dissipated at a unit area on the wetted perimeter. They gave the concept which states that the direction of energy transportation will define a minimum relative distance between the source of energy in the flow field and the boundary.

Shiono, Muto and Knight (1999) - presented the experimental data of secondary flow and turbulence using two components Laser- Doppler Anemometer for both straight and meandering channels to understand the flow mechanism in meandering channels. They developed *turbulence models* and studied the behaviour of secondary flow and centrifugal forces for both in-bank and over-bank flow conditions. They investigated the energy loss due to boundary friction, secondary flow, turbulence, expansion and contraction in meandering channels.

Ervine, Alan, Koopaei and Sellin (2000)- presented an analytical solution to the depth-integrated turbulent form of the Navier-Stokes equation that includes lateral shear and secondary flows in addition to bed friction. They applied this analytical solution to a number of channels, at model, and field scales, and predicted depth averaged velocity and shear stress for straight and meandering over bank.

Patra and Kar (2000) - used five dimensionless channel parameters to form equations representing the total shear force percentage carried by floodplains. They proposed a *variable-inclined interface* for which apparent shear force was calculated as zero. They even presented empirical equations for predicting proportion of discharge carried by the main channel and floodplain.

Sterling and Knight (2002)- used Shannon's (1948) *entropy concept* to predict the shear stress distribution on the wetted perimeter of a transverse section. They used two Langrangian multiplier, one for bed and other for the walls, for maximizing their entropy model. It was based on the hypothesis that in an open channel there are two certain regions in a cross section influencing the bed and wall respectively. Later in their study it was shown, the approach is roughly agreeable for circular channels with flatbed but could be of use if average trend value is of interest. They even stated the method predicts well for higher water depths than lower one but again when secondary current comes into play there would be a shift of max velocity from centerline which would lead to misleading prediction.

Yang and McCorquodale (2004) -developed a method for computing 3D Reynolds shear stresses and boundary shear stress distribution in smooth rectangular channels by applying an order of magnitude analysis to integrate the Reynolds equations. A simplified relationship between the lateral and vertical terms was hypothesized for which the Reynolds equations become solvable. This relationship was in the form of a power law with an exponent of n = 1, 2, or infinity. The semi-empirical equations for the boundary shear distribution and the distribution of Reynolds shear stresses were compared with measured data in open channels. The power-law exponent of 2 gave the best overall results while infinity gave good results near the boundary.

Dey and Lambert (2005) -derived a theoretical model using 2D Reynolds equation and continuity equation to compute the Reynolds stress and bed shear stress for a non-uniform unsteady nature of flow in a channel. They used the logarithmic law of velocity distribution and mixing length theory to derive the expressions for the mentioned purpose.

Guo and Julien (2005) - used continuity and momentum equation for a steady uniform flow to derive shear stress equations for wall and bed. Their approach was presented in two different approximations, first neglecting secondary current with constant eddy viscosity and in second empirical correction factors were used to take into account the effects of secondary current and variable eddy viscosity. *Conformal Mapping* was used to delineate the flow area into sub section based on hydraulic separation method as outlined by Leighly (1932) and Einstein (1942). These subsections were used to determine the shear stress by means of the derived equations for a rectangular straight channel. The second approximation gave an average relative error of 5.6 % having correlation coefficient of 0.994, while the first approximation underestimates the side wall shear stress.

Yu and Tan (2007) - used the shear stress equation proposed by Parker (1989). Assuming a 2D flow where the energy dissipates in the direction perpendicular the flow velocity isovels they proposed a method to estimate boundary shear stress using flownet, for a straight open channel either of circular, trapezoidal, rectangular or compound cross section. The model was based on discretising a set of differential equation with stream and potential functions, using a boundary fitted coordinate system over the wetted cross section. The flow net so mapped divides the flow area into various sub areas, each enclosed by two potential lines and a fraction of the wetted perimeter to which the area dissipates its energy. The effect of secondary flow was not considered, hence the method over estimates the value at sharp corners or protrusion. Else it showed a fair result for rectangular except at the toes, an error of around 6% for circular, for trapezoidal and compound the method estimates better than the Merged Perpendicular Method given by Khodashenas and Paquier (1999).

Khatua (2008) - extended the work of Patra and Kar (2000) to meandering compound channels and presented a general equations representing the *total shear force percentage* carried by floodplain.

Sclafani (2008) - carried out *Preston tube calibration* and obtained a close relationship between differential pressure and bed shear stress.

Khatua and Patra (2012)-developed a mathematical model using dimension analysis by taking series of experiments data to evaluate roughness coefficients for smooth and rigid meandering channels. The vital variables are required for stage-discharge relationship such as velocity, hydraulic radius, viscosity, gravitational acceleration, bed slope, sinuosity, and aspect ratio.

Pattnaik (2012) - investigated the effect of sinuosity and channel aspect ratio on the boundary shear in a meandering channel and proposed an equation to estimate *wall shear forces* for simple meandering with higher aspect ratio.

Samani, Farshi and Chamani (2013) -based on Guo and Julien (2005) study, provided somewhat similar expressions to determine the actual fraction of the field contribution to bed and wall shear for a trapezoidal channel. The work was based on conformal mapping and Schwarz-Christoffel transformation of the isovels and their orthogonals.

3.3 Critical Review

The research papers by Leighly (1932), Guo and Julien (2005), Samani, Farshi and Chamani (2013) concerns on isovels pattern of dividing the flow field of a straight channel to compute boundary shear stress. But the main drawback is that, firstly, it requires an extensive experimental study and, secondly, it is particularly difficult to employ it in a meandering channel where the isovels are distributed in a quite eccentric manner.

This implies us to go for the surplus energy approach which could be relatively simpler to employ. As Yang and Lim (1997, 1998) clearly explained the theory and its application, the main concern would be to extend the concept to meandering channels. In their method the distribution of boundary shear is uniform for the both walls which is unlikely to happen in a meandering channel as considerable curvature effects the flow. And even from any experimental data sets on boundary shear of a meandering channel it can be proven the same. So to incorporate the varying distribution, the effect of centrifugal force acting in a curve should be considered.

CHAPTER IV

BOUNDARY SHEAR STRESS DISTRIBUTION MODEL

4.1 Overview

This chapter contains the theory and assumptions behind the method developed to predict the boundary shear stress on the inner wall, outer wall and bed of a cross-section of a meandering channel at its bend apex. The flow of water in channels is generally governed by the Reynolds-Averaged Navier–Stokes (RANS) Equations (Schlichting 1979). RANS equation in a Cartesian coordinate system is used in order of magnitude analysis to seek the relative dependency and weightage of different parameter in the said problem. A meandering channel is characterised by substantial curvature and the equation used should be of curvilinear coordinate system, but for attaining relative ease in modelling, the Cartesian coordinate system is adopted with an assumption that, there won't be any notable difference. A model will be established to compute the shear stresses on the respective boundaries. As it is clear a model would be very difficult to be established based on only theory. The proposed model is established based on the energy balance of turbulent flow in a channel..

4.2 Theoretical Consideration

4.2.1 Reynolds-Averaged Navier-Stokes Equation

The Reynolds-Averaged Navier-Stokes equations for an incompressible fluid flow in the Cartesian coordinate system can be expressed as-

$$\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}+\frac{\partial \bar{u}}{\partial t}=g_x-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\upsilon\nabla^2\bar{u}-\left(\frac{\partial \overline{u'u'}}{\partial x}+\frac{\partial \overline{u'v'}}{\partial y}+\frac{\partial \overline{u'w'}}{\partial z}\right) \quad [4.1]$$

$$\bar{u}\frac{\partial \bar{v}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial y}+\bar{w}\frac{\partial \bar{v}}{\partial z}+\frac{\partial \bar{v}}{\partial t}=g_y-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y}+\upsilon\nabla^2\bar{v}-\left(\frac{\partial \overline{v'u'}}{\partial x}+\frac{\partial \overline{v'v'}}{\partial y}+\frac{\partial \overline{v'w'}}{\partial z}\right) \quad [4.2]$$

$$\bar{u}\frac{\partial \bar{w}}{\partial x}+\bar{v}\frac{\partial \bar{w}}{\partial y}+\bar{w}\frac{\partial \bar{w}}{\partial z}+\frac{\partial \bar{w}}{\partial t}=g_z-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z}+\upsilon\nabla^2\bar{w}-\left(\frac{\partial \overline{w'u'}}{\partial x}+\frac{\partial \overline{w'v'}}{\partial y}+\frac{\partial \overline{w'w'}}{\partial z}\right) \quad [4.3]$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the time-averaged velocity components while $\bar{p}$ is the time-averaged pressure intensity; and u', v' and w' are the fluctuations of u, v and w in x,y, and z direction respectively; and similarly instantaneous pressure, p, too has a fluctuating component, p'. Here u, v and w represent the instantaneous velocities at a point in the Cartesian coordinate system (x,y,z) which can be expressed as;

$$u = \bar{u} + u', \qquad v = \bar{v} + v', \qquad w = \bar{w} + w' \qquad \text{and} \qquad p = \bar{p} + p' \qquad [4.4]$$

The time-averaged value of a hydrodynamic quantity, say u, is given by

$$\bar{u} = \frac{1}{t_\infty} \int_{t_o}^{t_o + t_\infty} u\,dt \qquad [4.5]$$

where t_0 is any arbitrary time and t_∞ is the time over which the averaging is performed which should be sufficiently long so that the fluctuating component could be zero.

The tensor form of the equation set [4.1] to [4.3] can be expressed as

$$\bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_i}{\partial t} = g_{x_i} - \frac{1}{\rho} \cdot \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j{}^2} - \frac{\partial \overline{u_i' u'_j}}{\partial x_j}$$

where g_x, g_y, and g_z are the body forces per unit mass in x, y ,z directions and ρ is the mass density of fluid. $\bar{u}$ represents the time averaged velocity and u' is its fluctuation in i or j direction. Here i and j denotes 1,2 or 3 which represent x, y,z direction respectively. ν is the usual denotation for kinematic viscosity.

4.2.2 Order of Magnitude Analysis

The entire set of Navier-Stokes equations is so complex that it is unlikely and unrevealing to search for the most general solutions.Approximations valid for certain specific circumstance are much more useful.To make systematic approximations it is necessary to have a procedure that helps us discern precisely what is small and what is not. A standard procedure is to first find the scales relevant to the problem at hand. Normalization by these scales leads to dimensionless parameters which represent the relative importance of various parts of the full equations. Depending on the magnitudes of these parameters, suitable approximations can be devised which can lead to answers that capture the essence of the problem.

'Order of Magnitude Analysis' or 'Scaling Analysis' is a powerful tool used in mathematical science to extract useful and fundamental information from equations with many terms. In scaling analysis one does not seek to find a solution to the mathematical model in the conventional sense, but rather to make order of magnitude estimates for the likely outcome of computational solutions (Dowell and Jaworski, 2011). The purpose here is to provide a benchmark for the expected results of computational studies and also to estimate the relative importance of various effects that could be included in the model.

To define a model for meandering channel for calculating the boundary shear stress the analytical approach carried out by Yang and Lim (1997) for steady uniform straight channel based on surplus energy theory is initially taken into account; but the model is further rationally modified to include the non-uniformity characteristics of flow due to existence of substantial curvature in the channel. Hence here we aim to derive a steady non-uniform model that can predict the boundary shear stress on the inner, bed and outer wall of a cross-section.

Starting with the tensor form of Reynolds-Averaged Navier-Stokes (RANS) equation,

$$\bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_i}{\partial t} = g_{x_i} - \frac{1}{\rho} . \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^{\,2}} - \frac{\partial \overline{u'_i u'_j}}{\partial x_j} \quad [4.6]$$

Multiplying $\bar{u}_i$ to the eq. [3.6], we get

$$\bar{u}_i \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = g_{x_i} \bar{u}_i - \frac{1}{\rho} . \frac{\partial \bar{p}}{\partial x_i} \bar{u}_i + \nu \bar{u}_i \frac{\partial^2 \bar{u}_i}{\partial x_j^{\,2}} - \bar{u}_i \frac{\partial \overline{u'_i u'_j}}{\partial x_j} \quad [4.7]$$

where the last two terms can be written as:

$$\nu \bar{u}_i \frac{\partial^2 \bar{u}_i}{\partial x_j^{\,2}} = \frac{\partial}{\partial x_j} \left(\bar{u}_i \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) - \nu \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_i}{\partial x_j} \quad [4.8]$$

and, $$\bar{u}_i \frac{\partial \overline{u'_i u'_j}}{\partial x_j} = \frac{\partial (\bar{u}_i \overline{u'_i u'_j})}{\partial x_j} - \overline{u'_i u'_j} \frac{\partial \bar{u}_i}{\partial x_j} \quad [4.9]$$

Now substituting eq. [4.8] and [4.9] in eq [4.7], the equation becomes;

$$\bar{u}_j \frac{\partial(\bar{u}_i^2/2)}{\partial x_j} = g_{x_i}\bar{u}_i - \frac{1}{\rho}.\frac{\partial \bar{p}}{\partial x_i}u_i + \frac{\partial}{\partial x_j}\left[\bar{u}_i\left(-\overline{u_i'u_j'} + \nu\frac{\partial \bar{u}_i}{\partial x_j}\right)\right] - \frac{\partial \bar{u}_i}{\partial x_j}\left(-\overline{u_i'u_j'} + \nu\frac{\partial \bar{u}_i}{\partial x_j}\right) \quad [4.10]$$

In a Cartesian coordinate system, shear stress is expressed as

$$\tau_{ij} = -\rho\overline{u_i'u_j'} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad [4.11]$$

So, the eq. [4.10], turns to

$$\bar{u}_j \frac{\partial(\bar{u}_i^2/2)}{\partial x_j} = g_{x_i}\bar{u}_i - \frac{1}{\rho}.\frac{\partial \bar{p}}{\partial x_i}u_i + \frac{1}{\rho}\frac{\partial(\bar{u}_i\tau_{ij})}{\partial x_j} - \frac{1}{\rho}\frac{\partial \bar{u}_i}{\partial x_j}(\tau_{ij}) \quad [4.12]$$

In the above eq. [4.12], the left side represents the change in kinetic energy per unit mass and is equal to the algebraic sum of the quantities at its right side where the first term represents the work done by the body force, the second term is the work done by pressure force, third is the rate of energy transferred and the last one is the rate of energy dissipated. The left term can be expressed as:

$$\bar{u}_j \frac{\partial(\bar{u}_i^2/2)}{\partial x_j} = \frac{\partial(\bar{u}_j\bar{u}_i^2/2)}{\partial x_j} - \frac{\bar{u}_i^2}{2}\frac{\partial \bar{u}_j}{\partial x_j} \quad [4.13]$$

The change in the depth averaged velocity is zero in the same direction of averaging. Therefore, $\frac{\partial \bar{u}_j}{\partial x_j} = 0$ and hence arranging the remaining terms,

$$\rho g_{x_i}\bar{u}_i - \frac{\partial \bar{p}}{\partial x_i}u_i + \frac{\partial}{\partial x_j}\left[\bar{u}_i\left(\tau_{ij} - \frac{\rho\bar{u}_i\bar{u}_j}{2}\right)\right] - \frac{\partial \bar{u}_i}{\partial x_j}(\tau_{ij}) = 0 \quad [4.14]$$

Or, $g_{x_i}\bar{u}_i - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i}u_i + \frac{\partial}{\partial x_j}\left[\bar{u}_i\left(\frac{\tau_{ij}}{\rho} - \frac{\bar{u}_i\bar{u}_j}{2}\right)\right] - \frac{\partial \bar{u}_i}{\partial x_j}\left(\frac{\tau_{ij}}{\rho}\right) = 0$ [4.15]

As the flow is non-uniform the rate of energy transfer in x direction won't be zero. As we are concerned about quantifying the shear stress energy, following the hypothetical assumption that, the shear stress at the wetted boundary is the surplus energy available at its wetted cross section. And this surplus energy gets transferred to the boundary by a certain path. Hence expanding only the third term, which is concerned with the energy transfer in a flow, we get:

$$E_t = \frac{\partial}{\partial x_j}\left[\bar{u}_i\left(\tau_{ij} - \frac{\rho \bar{u}_i \bar{u}_j}{2}\right)\right] = \frac{\partial}{\partial x}\left[\bar{u}\left(\frac{\tau_{xx}}{\rho} - \frac{\bar{u}^2}{2}\right) + \bar{v}\left(\frac{\tau_{yx}}{\rho} - \frac{\bar{v}\bar{u}}{2}\right) + \bar{w}\left(\frac{\tau_{zx}}{\rho} - \frac{\bar{w}\bar{u}}{2}\right)\right]$$
$$+ \frac{\partial}{\partial y}\left[\bar{u}\left(\frac{\tau_{xy}}{\rho} - \frac{\bar{u}\bar{v}}{2}\right) + \bar{v}\left(\frac{\tau_{yy}}{\rho} - \frac{\bar{v}^2}{2}\right) + \bar{w}\left(\frac{\tau_{zy}}{\rho} - \frac{\bar{w}\bar{v}}{2}\right)\right]$$
$$+ \frac{\partial}{\partial z}\left[\bar{u}\left(\frac{\tau_{xz}}{\rho} - \frac{\bar{u}\bar{w}}{2}\right) + \bar{v}\left(\frac{\tau_{yz}}{\rho} - \frac{\bar{v}\bar{w}}{2}\right) + \bar{w}\left(\frac{\tau_{zz}}{\rho} - \frac{\bar{w}^2}{2}\right)\right] \qquad [4.16]$$

Now to find the direction to which energy is transferred, '*Order of Magnitude Analysis*' as followed by Yang and Lim [1997] will be carried out.

Let L_x, L_y, L_z denotes w.r.t x, y and z direction respectively. Assuming the shear stress τ_{xy} , τ_{yy} , τ_{xz} and τ_{zz} are of same order they are represented as τ for analysis, that means the ratio between any two would be one or less than one. This assumption is proved correct at the later of this section. So a term is represented as-

$$\frac{\partial}{\partial x}\left(\bar{u}\,\tau_{xx}\right) = O\left(\frac{u\tau}{L_x}\right)$$

where "O" represent "of the order of". Then eq. [4.16] would be

$$E_t = \frac{\partial}{\partial x_j}\left[\bar{u}_i\left(\tau_{ij} - \frac{\rho \bar{u}_i \bar{u}_j}{2}\right)\right] = \frac{u\tau_{xx}}{L_x};\frac{u^3}{L_x};\frac{v\tau_{yx}}{L_x};\frac{v^2u}{L_x};\frac{w\tau_{zx}}{L_x};\frac{w^2u}{L_x};$$
$$\frac{u\tau}{L_y};\frac{u^2v}{L_y};\frac{v\tau}{L_y};\frac{v^3}{L_y};\frac{w\tau}{L_y};\frac{w^2v}{L_y};$$
$$\frac{u\tau}{L_z};\frac{u^2w}{L_z};\frac{v\tau}{L_z};\frac{v^2w}{L_z};\frac{w\tau}{L_z};\frac{w^3}{L_z} \qquad [4.17]$$

Hence the simplified form of eq. [4.17] is

$$E_t = \frac{\partial}{\partial x_j}\left[\bar{u}_i\left(\tau_{ij} - \frac{\rho \bar{u}_i \bar{u}_j}{2}\right)\right] = \frac{\partial}{\partial x}\left[\bar{u}\left(\frac{\tau_{xx}}{\rho} - \frac{\bar{u}^2}{2}\right)\right] + \frac{\partial}{\partial y}\left[\bar{u}\left(\frac{\tau_{xy}}{\rho} - \frac{\bar{u}\bar{v}}{2}\right)\right] + \frac{\partial}{\partial z}\left[\bar{u}\left(\frac{\tau_{xz}}{\rho} - \frac{\bar{u}\bar{w}}{2}\right)\right]$$

$$= \frac{u\tau_{xx}}{L_x};\frac{u^3}{L_x};\frac{u\tau_{xy}}{L_y};\frac{u^2 v}{L_y};\frac{u\tau_{xz}}{L_z};\frac{u^2 w}{L_z} \qquad [4.18]$$

or,

$$;\frac{u}{L_x};\frac{u^2}{\tau_{xx}};\frac{\tau_{xy}}{\tau_{xx}}\frac{L_x}{L_y};\frac{uv}{\tau_{xx}}\frac{L_x}{L_y};\frac{\tau_{xz}}{\tau_{xx}}\frac{L_x}{L_z};\frac{uw}{\tau_{xx}}\frac{L_x}{L_z} \qquad [4.19]$$

or,

$$;\frac{\tau_{xx}}{\tau_{xy}}\frac{L_y}{L_x};\frac{u^2}{\tau_{xy}}\frac{L_y}{L_x};1;\frac{uv}{\tau_{xy}};\frac{\tau_{xz}}{\tau_{xy}}\frac{L_y}{L_z};\frac{uw}{\tau_{xy}}\frac{L_y}{L_z} \qquad [4.20]$$

From the equation [4.20], it can be stated energy is transferred in all three direction; but on close observation it can be inferred τ_{xx} which is nothing but the stress in normal direction and is the energy along the flow. While the stress τ_{xy} and τ_{xz} are the shear stresses or the resistance parallel to boundary. Let us assume that the energy that flows in x direction into a section is equal to the energy that flows out of that section. At later of this section this assumption will be taken into account and hence the term which is actually concerned for the boundary will be in y and z direction. Therefore, the approximate energy transfer for boundary shear would be,

$$E_t = \frac{\partial}{\partial y}\left[\bar{u}\left(\frac{\tau_{xy}}{\rho} - \frac{\bar{u}\bar{v}}{2}\right)\right] + \frac{\partial}{\partial z}\left[\bar{u}\left(\frac{\tau_{xz}}{\rho} - \frac{\bar{u}\bar{w}}{2}\right)\right];\frac{u\tau_{xy}}{L_y};\frac{u^2 v}{L_y};\frac{u\tau_{xz}}{L_z};\frac{u^2 w}{L_z};1;\frac{uv}{\tau_{xy}};\frac{\tau_{xz}}{\tau_{xy}}\frac{L_y}{L_z};\frac{uw}{\tau_{xy}}\frac{L_y}{L_z} \qquad [4.21]$$

Now neglecting the viscosity effect in the main flow, the shear stress can be expressed as $-\rho\overline{u_i' u_j'}$.

Then,

$$\frac{\tau_{xz}}{\tau_{xy}} = \frac{\overline{u'w'}}{\overrightarrow{u'v'}} \propto \left(\frac{\bar{u}'^2\,\bar{w}'^2}{\bar{u}'^2\,\bar{v}'^2}\right)^{1/2} \propto \left(\frac{\bar{w}'^2}{\bar{v}'^2}\right)^{1/2} \qquad [4.22]$$

$$\frac{\tau_{xx}}{\tau_{xy}} = \frac{\overline{u'u'}}{\overrightarrow{u'v'}} \propto \left(\frac{\bar{u}'^2\,\bar{u}'^2}{\bar{u}'^2\,\bar{v}'^2}\right)^{1/2} \propto \left(\frac{\bar{u}'^2}{\bar{v}'^2}\right)^{1/2} \qquad [4.23]$$

$$\frac{\tau_{yy}}{\tau_{xy}} = \frac{\overline{v'v'}}{\overrightarrow{u'v'}} \propto \left(\frac{\bar{v}'^2\,\bar{v}'^2}{\bar{u}'^2\,\bar{v}'^2}\right)^{1/2} \propto \left(\frac{\bar{v}'^2}{\bar{u}'^2}\right)^{1/2} \qquad [4.24]$$

Hence the $O\left(\frac{\tau_{xz}}{\tau_{xy}}\right) \leq 1$, $O\left(\frac{\tau_{xx}}{\tau_{xy}}\right) > 1$ and $O\left(\frac{\tau_{yy}}{\tau_{xy}}\right) \leq 1$. Additionally, Tracy's (1965) results showed that the highest turbulence level occurs at the boundary, and then decreases with increasing distance from the boundary. Therefore the τ_{xz}/τ_{xy} can be written as;

$$\frac{\tau_{xz}}{\tau_{xy}} \cong \left(\frac{\overline{w}'^2}{\overline{v}'^2}\right)^{1/2} \propto \frac{L_y}{L_z} \qquad [4.25]$$

and hence when $\frac{L_y}{L_z}$ <<1, the term with $\frac{\partial}{\partial z}$ in eq. [4.21] can be ignored. And the surplus energy only gets transferred toward the y path i.e towards the bed. The converse, when $\frac{L_z}{L_y}$ <<1, $\frac{\partial}{\partial y}$ is ignored and energy transfers towards the wall. Here L_y and L_z are the normal distance towards the boundary. But again, in a meandering channel as the two sides wall resists unequal shear stress, there need to be a condition to find whether the distance towards the outer or inner wall is minimal.

In a steady non-uniform flow, the non-uniformity is characterized by varied depth and varied mean velocity with respect to space which is due to difference in bottom slope and energy slope in the flow direction. Two certain cases are possible; either the depth increases or decreases in the flow direction. And in a meandering channel both of the cases are observed. Hence the slope of the energy line is considered as a constant value, $S_f=\Delta H_L/L$, for a stretch of length, L with difference in mean water depth at the starting and ending section of the stretch as ΔH_L.

4.3 Analytical Formulation

The above section, can be interpreted as, the surplus energy from a flow section dissipates on the boundary in the form of shear stress at a cross-section by following a proximal distance. This also been observed by (Chien and Wan, 1986), that the surplus energy in the main flow transfers and eventually dissipated as heat at the boundary. Experimental data shows the shear stresses in the inner wall or bank is subjected to more shear stress than the inner wall. This clearly suggests the region from the flow area contributing energy to the outer bank is more than the inner bank to balance energy at a section. Then again bed of a channel also takes a considerable amount of energy and hence the flow area need to be divided into three sub-regions for hydraulic radius computation to calculate the shear stress on inner wall, outer wall and bed based on conventional shear stress formulae ($\tau = \rho gRS$) with constant energy slope but different hydraulic radius for the three regions.

Hence our next step is to define a way to delineate the flow region into three sub-regions by division lines justifying the logic as explained. And then validating the existence of this division line by experimental data. To define this division line certain parameters pertaining to position in the flow field, surface condition of the boundary and curvature are considered which are explained in the following sections in detail.

4.3.1 Parameters Considered

Three parameters are considered for modelling designating geometrical distance, surface condition and curvature effect respectively. These three parameters are used to define relative distance which for elemental volume through which the excess energy will be dissipated.

4.3.1.1 Geometrical distance

The direction of energy transportation is along the shortest geometrical distance, 'l', between the location of the energy source concerned and the boundary. An element in a flow field as shown in fig.[4.1] have three possible shortest path through which the surplus energy can travel to its boundary. Then the geometrical distance, ψ, would be the least among all the three, i.e.

Geometrical Distance, ψ =Minimum {a, b, c}

It may be noted that the energy dissipation is not concerned at a point but on certain region on the boundary which is nearest to the energy source. (proximal distance)

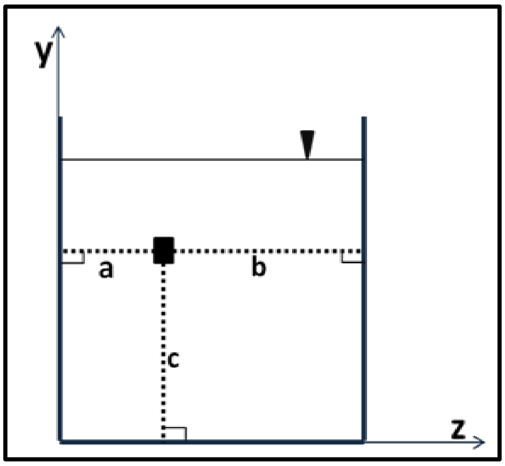


Fig.[4.1]- Possible shortest distance to boundaries

4.3.1.2 Surface condition

The energy dissipation capacity of the boundary has an indirect influence on the amount of energy that will be transported from the flow field, Yang-Lim (1997). Based on the friction experienced at its boundary the energy dissipation varies. It Means a rough channel surface will dissipate more than a smoother one.

- **<u>Smooth Surface-</u>** Here friction depends on Reynolds number, 'Re', based on the mean flow velocity, and the viscous force. Hence thickness of viscous sub-layer, D_s, should be used as the characteristic length, D. The viscous sub-layer is defined as, $u^*D_s/\nu=C_s$= constant and hence

$$D = \frac{D_s}{C_s} = \frac{u_*}{\nu} \qquad [4.26]$$

- **<u>Rough Surface-</u>** Here the friction depends mainly on the boundary roughness. The viscous effect in this case can be neglected, and the characteristic length *D* can be assumed as the roughness height. Hence,

$$D = D_r = \Delta \qquad [4.27]$$

4.3.2 Curvature

Centrifugal force acts in a radial direction when flow enters a curve. It is expressed as, WV_m^2/gR_m Where W is the weight of fluid element, V_m is the mean flow velocity, g is acceleration due to gravity and is the mean radius of the channel. Then force per unit weight would be V_m^2/gR_m .

Work done per unit weight of the fluid towards its boundary= $\dfrac{V_m^2}{gR_m}\psi$ [4.28]

Where, ψ is the geometrical distance of the unit weight of fluid to its nearest boundary

4.3.3 Relative Distance

To bring together all three parameters explained in the previous section, a proposed dimensionless parameter called "Relative distance", Φ, is defined as-

For walls, $$\phi_w = \frac{\psi \pm \dfrac{V_m^2}{gR_m}\psi}{D} = \frac{\psi}{D}\left[1 \pm \frac{V_m^2}{gR_m}\right] = \frac{\psi}{D}\left[1 \pm C\right]$$ [4.29]

And for bed, $$\phi_b = \frac{\psi}{D}$$ [4.30]

Here, + is used when work done is along the direction of centrifugal force and – when it is against the direction of force.

4.4 Rectangular Channel

Consider the case of a steady, uniform flow in a smooth rectangular channel with an aspect ratio, α=b/h, where h = flow depth and b =channel width. The side wall and bed will each have its own fair share of surplus energy transferred from the main flow, depending on the minimum relative distance given by [4.29]. Three cases arise as per the flow depth- i) $h<h_{cr}$ ii) $h>h_{cr}$ and iii) $h=h_{cr}$, where h_{cr} = depth of water at critical condition, which will be explained later in detail.

4.4.1 [Case I]: $\alpha \geq \alpha_{cr}$

Let us consider the left wall to be the inner wall of a meander and the right to be the outer. In this case, for any unit volume in the flow field, there are three possible ways to transfer the surplus energy, i.e., either toward the bed ($\Phi_b>\Phi_{wi}$ or $\Phi_b>\Phi_{wo}$) for region aoc as shown in fig [3.4] or toward the inner sidewall ($\Phi_{wi}>\Phi_b$) for region aeg or towards outer sidewall for region cdi ($\Phi_{wo}>\Phi_b$). If $\Phi_{wi}>\Phi_b$ or $\Phi_{wo}>\Phi_b$, the energy will be transferred toward the left or right side wall respectively. Conversely, if $\Phi_b>\Phi_{wi}$ or $\Phi_b>\Phi_{wo}$ then the energy will be transported to the bed. It follows the conditions where, $\phi_{wi}=\phi_b$ and $\phi_{wo}=\phi_b$, will define the division line. Then,

a) **Division line between Inner wall and Bed**. i.e. for line ae,

$$\phi_{wi}=\phi_b$$

From eq [4.4], relative distance can be substituted as,

or, $$\frac{z}{D_{wi}}[1-C]=\frac{y}{D_b}$$

or, $$z=k_1\frac{y}{[1-C]} \qquad [4.31]$$

$$k_1=\frac{u_b^*}{u_{wi}^*} \qquad [4.32]$$

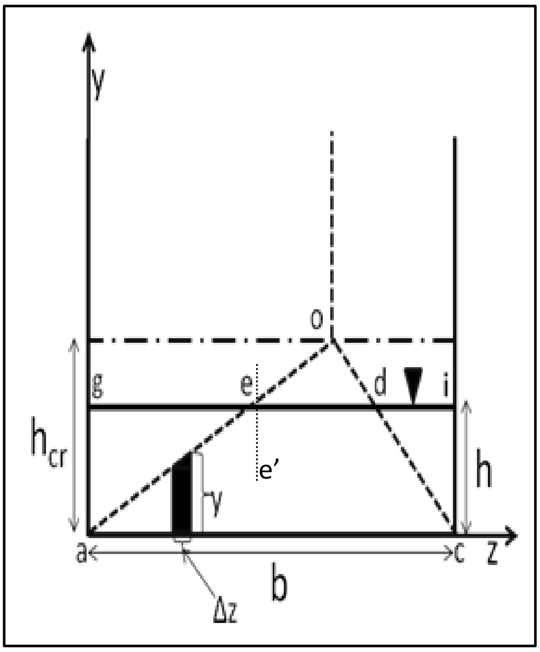


Fig.[4.2] For $\alpha \geq \alpha_{cr}$

Here z= least distance if the points lying on line ae to inner wall.

y= least distance if the points lying on line ae to bed.

$$C=\frac{V_m^2}{gR_m}$$, as explained earlier.

D_{wi} and D_b represents the boundary condition of inner wall and bed respectively.

$u*_b$ and $u*_{wi}$ are the shear velocity

b) Division line between Outer wall and Bed i.e. for line cd,

$\phi_{wo} = \phi_b$ and then substituting, we get

$$\frac{b-z}{D_{wo}}[1+C]=\frac{y}{D_b}$$

or, $$b-z=k_2\frac{y}{[1+C]} \qquad [4.33]$$

$$k_2=\frac{u_b^*}{u_{wo}^*} \qquad [4.34]$$

Here, z is the perpendicular distance from the left wall and b-z is the least perpendicular distance to right wall.

Area for an element within region aee', as shown in fig [4.2], $\Delta A_b = y\,\Delta z$

Shear stress on Δz area for region aee', $\Delta\tau_b = \rho g(\Delta A/\Delta P)S_o = \rho g y S_o$

So the average shear stress on bed,

$$\overline{\tau_b}=\begin{cases}\rho g\dfrac{z(1-c)}{k_1} & 0\le z\le k_1h/(1-C)\\ \rho g h S & k_1h/(1-C)\le z\le b-(k_2h/1+C)\\ \rho g\dfrac{(b-z)(1+c)}{k_2}S & b-(k_2h/1+C)\le z\le b\end{cases}$$

or, $$\overline{\tau_b}=\frac{1}{b}\left[\int_0^{\frac{k_1h}{1-C}}\rho g\frac{z(1-c)}{k_1}Sdz+\int_{\frac{k_1h}{1-C}}^{b-\frac{k_2h}{1+C}}\rho ghSdz+\int_{b-\frac{k_2h}{1+C}}^{b}\rho g\frac{(b-z)(1+c)}{k_2}Sdz\right]$$

or, $$\overline{\tau_b} = \frac{\rho g S h}{b}\left[b - \frac{h}{2}\left(\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)} \right) \right] \qquad [4.35]$$

Similarly, ΔA_w=z (Δy) and $\Delta\tau_w$= ρg(ΔA/ΔP)S_o = ρgzS_o $\overline{\tau_{wi}} = \frac{1}{h}\left[\int_0^h \rho g k_1 \frac{y}{(1-C)} S \right]$

or, $$\overline{\tau_{wi}} = \rho g S h \frac{k_1}{2(1-C)} \qquad [4.36]$$

and $$\overline{\tau_{wo}} = \frac{1}{h}\left[\int_0^h \rho g k_2 \frac{y}{(1+C)} S \right]$$

or, $$\overline{\tau_{wo}} = \rho g S h \frac{k_2}{2(1+C)} \qquad [4.37]$$

Equation [4.35], [4.36] and [4.37] will give the average shear stress for bed, inner wall and outer wall respectively.

we have $\tau_0 = u_*^{\ 2}\rho$. Hence using equations [4.35] and [4.36]

$$\frac{\overline{\tau_b}}{\overline{\tau_{wi}}} = \left(\frac{u_b^*}{u_{wi}^*} \right)^2 = k_1^{\ 2} = \frac{\frac{\rho g S h}{b}\left[b - \frac{h}{2}\left(\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)} \right) \right]}{\rho g S h \frac{k_1}{2(1-C)}}$$

or, $$\frac{b}{h}\frac{k_1^{\ 3}}{2(1-C)} + \frac{k_1}{2(1-C)} + \frac{k_2}{2(1+C)} - \frac{b}{h} = 0 \qquad [4.38]$$

Similarly using eq. [4.36] and [4.37]

$$\frac{\overline{\tau_b}}{\overline{\tau_{wo}}} = \left(\frac{u_b^*}{u_{wo}^*} \right)^2 = k_2^{\ 2} = \frac{\frac{\rho g S h}{b}\left[b - \frac{h}{2}\left(\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)} \right) \right]}{\rho g S h \frac{k_2}{2(1+C)}}$$

or, $$\frac{b}{h}\frac{k_2^{\ 3}}{2(1+C)} + \frac{k_1}{2(1-C)} + \frac{k_2}{2(1+C)} - \frac{b}{h} = 0 \qquad [4.39]$$

Solving eq. [4.38] and [4.39], we get,

$$\left[\frac{k_1}{k_2}\right] = \left[\frac{1-C}{1+C}\right]^{1/3} \qquad [4.40]$$

As $k_1/k_2=u^*_{wo}/u^*_{wi}$, the above equation 4.40 gives a relation between the shear velocities for inner and outer wall for a particular water depth due to effect of centrifugal force unlike in a straight channel, where both are equal.

Using equation 4.38 and 4.39 k_1 and k_2 can be determined for a water depth and this can be used to compute average shear stress acting on walls and bed in eq 4.35, 4.36 and 4.37.

4.4.2 [Case II]: $\alpha = \alpha_{cr}$

At critical depth $\alpha=\alpha_{cr}$ and $h=h_{cr}$. Hence for point O in the flow field, the condition it should satisfy is,

$$\phi_{wi} = \phi_b = \phi_{wo}$$

Which, after substitution reduces to as,

$$\frac{b}{h_{cr}} = \frac{k_1}{(1-C)} + \frac{k_2}{(1+C)} \qquad [4.41]$$

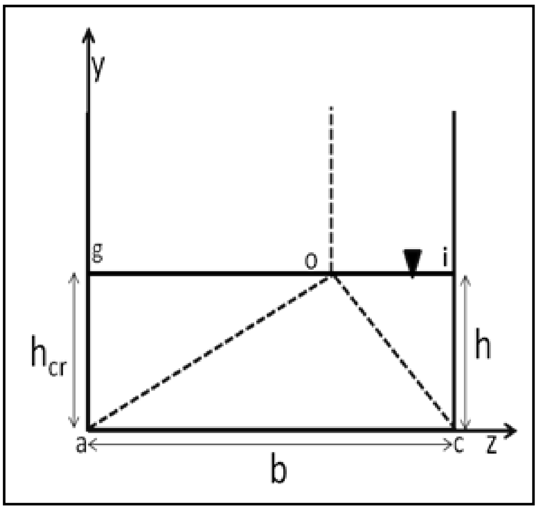


Fig [4.3] At Critical Condition

<u>Determination of Critical Depth, h_{cr} of a channel</u>

Using eq [4.16] with [4.13] and [4.14], the given equation can be derived-

$$\frac{b}{h_{cr}} = (1-C)^{-2/3} + (1+C)^{-2/3}$$

or, $$h_{cr} = \frac{b}{(1-C)^{-2/3} + (1+C)^{-2/3}} \qquad [4.42]$$

Therefore, critical depth is a function of channel width and centrifugal ratio. Here to compute centrifugal ratio, C, mean velocity and mean radius of channel is required. Manning's equation to compute velocity is used.

4.4.3 [Case III]: $\alpha \le \alpha_{cr}$

In a similar manner for $\alpha \le \alpha_{cr}$, $h>h_{cr}$ and hence flow field division can be as shown in fig [4.4]. So,

a) Division line between Inner wall and Bed. i.e. for line ae

b) Division line between Outer wall and Bed i.e. for line cd,

Equation for line ao and co will be in same as equation [4.33] and [4.35] respectively. Using either of the equation with $y=h_{cr}$, we can get

$$h_{cr} = \frac{b(1+C)(1-C)}{k_1(1+C)+k_2(1-C)} \qquad [4.43]$$

c) Division line between Inner and Outer wall i.e. for line ok

$$\phi_{wi} = \phi_{wo}$$

or, $$\frac{z}{D_{wi}}[1-C] = \frac{b-z}{D_{wo}}[1+C]$$

or, $$z = \frac{bk_1[1+C]}{k_1[1+C]+k_2[1-C]} \qquad [4.44]$$

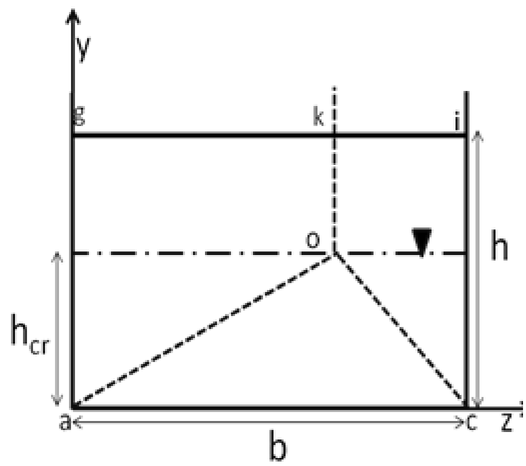


Fig.[4.4] $\alpha \le \alpha_{cr}$

Here we can go on integrating in same manner or determine the respective areas using equation [4.43] and [4.44]

$$A_{wi} = hz - \frac{1}{2}zh_{cr} = z[h - \frac{h_{cr}}{2}] = \frac{bk_1[1+C]}{k_1[1+C]+k_2[1-C]}\left[h - \frac{1}{2}\left(\frac{b(1+C)(1-C)}{k_1(1+C)+k_2(1-C)}\right)\right]$$

$$A_{wb} = \frac{1}{2}bh_{cr} = \frac{1}{2}b\left(\frac{b(1+C)(1-C)}{k_1(1+C)+k_2(1-C)}\right)$$

$$A_{wo} = bh - A_{wi} - A_{wb} = h(b-z) - \frac{1}{2}(b-z)h_{cr}$$

And then average shear stress can be computed as-

$$\overline{\tau_b} = \rho g \frac{A_{wb}}{b} S = \frac{1}{2}\left(\frac{\rho g S b(1+C)(1-C)}{k_1(1+C)+k_2(1-C)}\right) = \frac{\rho g S b}{2}\left(\frac{1}{\frac{k_1}{(1-C)}+\frac{k_2}{(1+C)}}\right) \qquad [4.45]$$

$$\bar{\tau}_{wi} = \rho g \frac{A_{wi}}{h} S = \rho g S \frac{b k_1[1+C]}{k_1[1+C]+k_2[1-C]}\left(1-\frac{b}{2h}\left(\frac{(1+C)(1-C)}{k_1(1+C)+k_2(1-C)}\right)\right)$$

or, $$\bar{\tau}_{wi} = \rho g S \frac{b k_1}{(1-C)}\left(\frac{1}{\frac{k_1}{[1-C]}+\frac{k_2}{[1+C]}}\right)\left[1-\frac{b}{2h}\left(\frac{1}{\frac{k_1}{[1-C]}+\frac{k_2}{[1+C]}}\right)\right] \qquad [4.46]$$

$$\bar{\tau}_{wo} = \rho g \frac{A_{wo}}{h} S = \bar{\tau} - \overline{\tau_b} - \overline{\tau_{wi}} \text{ where } \bar{\tau} = \rho g R S$$

or, $$\bar{\tau}_{wo} = \rho g S \frac{b k_2}{(1+C)}\left(\frac{1}{\frac{k_1}{[1-C]}+\frac{k_2}{[1+C]}}\right)\left[1-\frac{b}{2h}\left(\frac{1}{\frac{k_1}{[1-C]}+\frac{k_2}{[1+C]}}\right)\right] \qquad [4.47]$$

Similarly, we have $\tau_0 = u_*^2 \rho$. Hence using equations [4.20] and [4.21]

$$\frac{\overline{\tau_b}}{\overline{\tau_{wi}}} = \left(\frac{u_b^*}{u_{wi}^*}\right)^2 = k_1^2 = \frac{\frac{\rho g S b}{2}\left(\frac{1}{\frac{k_1}{(1-C)}+\frac{k_2}{(1+C)}}\right)}{\rho g S \frac{b k_1}{(1-C)}\left(\frac{1}{\frac{k_1}{[1-C]}+\frac{k_2}{[1+C]}}\right)\left[1-\frac{b}{2h}\left(\frac{1}{\frac{k_1}{[1-C]}+\frac{k_2}{[1+C]}}\right)\right]}$$

Simplifying, $$\frac{2k_1^3}{(1-C)}\left[1-\frac{b}{2h}\left(\frac{1}{\frac{k_1}{(1-C)}+\frac{k_2}{(1+C)}}\right)\right] = 1 \qquad [4.48]$$

Similarly using eq [4.46] and [4.47], we get

$$\frac{\overline{\tau_b}}{\tau_{wo}} = \left(\frac{u_b^*}{u_{wo}^*}\right)^2 = k_2^{\,2} = \frac{\dfrac{\rho g S b}{2}\left(\dfrac{1}{\dfrac{k_1}{(1-C)}+\dfrac{k_2}{(1+C)}}\right)}{\rho g S\dfrac{bk_2}{(1+C)}\left(\dfrac{1}{\dfrac{k_1}{[1-C]}+\dfrac{k_2}{[1+C]}}\right)\left[1-\dfrac{b}{2h}\left(\dfrac{1}{\dfrac{k_1}{[1-C]}+\dfrac{k_2}{[1+C]}}\right)\right]}$$

Simplifying, $$\frac{2k_2^3}{(1+C)}\left[1-\frac{b}{2h}\left(\frac{1}{\dfrac{k_1}{(1-C)}+\dfrac{k_2}{(1+C)}}\right)\right]=1 \qquad [4.49]$$

Equating L.H.S of eq.[4.48] and [4.49], we get

$$\left[\frac{k_1}{k_2}\right]=\left[\frac{1-C}{1+C}\right]^{1/3} \qquad [4.50]$$

which is same as eq[4.40] of case I.

Here we need to solve eq [4.48] qnd [4.49] to get k1 and k2. Then use equation [4.45]-[4.47] to determine stresses.

4.5 Trapezoidal Channel

As per the theoretical consideration followed in the previous section, this section concerns formulation for a trapezoidal channel. Unlike rectangular, the trapezoidal channel has slight variation while dividing the cross section. Let us consider a symmetric trapezoidal channel having channel bottom width as b and a flow depth of ‘h’. The side slope of the channel is defined by the angle ‘θ’ inclined to the horizontal as shown the fig [4.5]. Based on the flow depth of water three different cases arises: [I] $\alpha \geq \alpha_{cr}$, [II] $\alpha = \alpha_{cr}$ [III] $\alpha \leq \alpha_{cr}$, where α is defined as b/h and α_{cr} as b/h_{cr}.

4.5.1 [Case I]: $\alpha \geq \alpha_{cr}$

In this condition the section can be divided as shown in fig [4.5]. The actual flow area is acgf. Each elemental flow area transfers the energy to its nearest or proximal boundary. Line ad and ce divides the flow area into three region as adf, aced and ecg which contributes energy to left wall, bed and right wall respectively. Let the left wall be the inner and right be the outer wall of a meander channel. Now to define the path of the line ad and ce, we need to equate the 'relative distance' of each point on this line to wall and bed as :

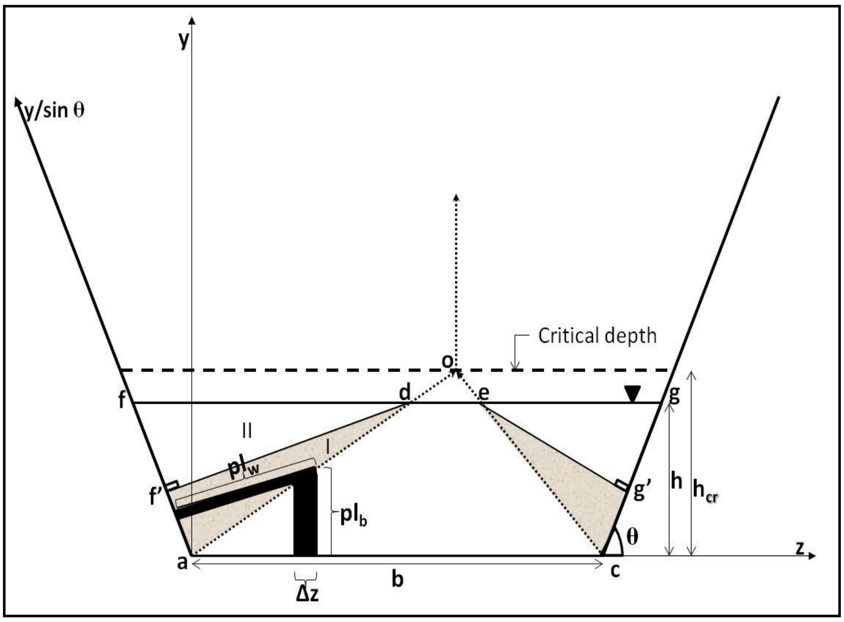


Fig [4.5]: For $\alpha \geq \alpha_{cr}$

a) Division line between wall and Bed. i.e. for line ad,

Each point on this line should have equal relative distance (Φ) to the inner wall and to the bed. The nearest point to which it can contribute energy be at a distance (proximal length) of pl_w to wall and pl_b to the bed. Then,

$$\phi_{wi} = \phi_b \qquad [4.51]$$

Substituting the parameter as explained in the section [4.2], we get

$$\frac{pl_{wi}}{D_{wi}}[1-C] = \frac{pl_b}{D_b} \qquad [4.52]$$

$$pl_{wi} = \frac{k_1 pl_b}{[1-C]} \qquad [4.53]$$

where, pl is the proximal length to the respective boundaries, C is the centrifugal ratio due to curvature of the channel and k_1, k_2 are the ratios between the boundary condition as explained earlier.

From Fig[4.5]

$$P_{wi} = z\sin\theta + y\cos\theta \tag{4.54}$$

$$P_b = y \tag{4.55}$$

which gives us the equation as,

$$z\sin\theta + y\cos\theta = \frac{k_1 y}{[1-C]} \tag{4.56}$$

solving we get, $z = y\left[\frac{k_1}{[1-C]} - \cos\theta\right]\frac{1}{\sin\theta}$ [4.57]

or, $z = y\left[\frac{k_1}{[1-C]\sin\theta} - \cot\theta\right]$ [4.58]

As y is the only variant and k_1, C, θ remains constant for a particular channel having same radius, the equation can be expressed in simpler manner as,

$$z = yC_1 \tag{4.59}$$

Where, $C_1 = \frac{k_1}{[1-C]\sin\theta} - \cot\theta$

b) **Division line between Outer wall and Bed** i.e. for line ce,

Similarly for line ce, the relative distance between the outer wall and the line (ϕ_{wi}) should be equated with the relative distance between bed and the line (ϕ_b),

$\phi_{wi} = \phi_b$ and on substituting and further solving results eq. [4.60].

$$\frac{pl_{wo}}{D_{wo}}[1+C] = \frac{pl_b}{D_b} \tag{4.60}$$

$$pl_{wo} = \frac{k_2 pl_b}{[1+C]} \qquad [4.61]$$

From fig.[4.5],

$$pl_{wo} = (b-z)\sin\theta + y\cos\theta \qquad [4.62]$$

$$pl_b = y \qquad [4.63]$$

Putting this, we get

$$(b-z)\sin\theta + y\cos\theta = \frac{k_2 y}{[1+C]} \qquad [4.64]$$

$$\text{or, } (b-z) = y\left[\frac{k_2}{[1+C]} - \cos\theta\right]\frac{1}{\sin\theta} \qquad [4.65]$$

$$\text{or, } (b-z) = y\left[\frac{k_{21}}{[1+C]\sin\theta} - \cot\theta\right] \qquad [4.66]$$

Finally, we get,

$$b - z = yC_2 \qquad [4.67]$$

$$\text{where, } C_2 = \frac{k_{21}}{[1+C]\sin\theta} - \cot\theta$$

Then the average shear stress on the bed is

$$\overline{\tau_b} = \begin{cases} \rho g \dfrac{z}{C_1} S & 0 \le z \le hC_1 \\ \rho g h S & hC_1 \le z \le b - hC_2 \\ \rho g \dfrac{(b-z)}{C_2} S & b - hC_2 \le z \le b \end{cases}$$

$$\Rightarrow \overline{\tau_b} = \frac{1}{b}[\int_0^{hC_1} \rho g \frac{z}{C_1} Sdz + \int_{hC_1}^{b-hC_2} \rho g h S dz + \int_{b-hC_2}^{b} \rho g \frac{(b-z)}{C_2} Sdz]$$

$$\Rightarrow \overline{\tau_b} = \frac{\rho g S h}{b}\left[b - \frac{h}{2}(C_1 + C_2)\right]$$

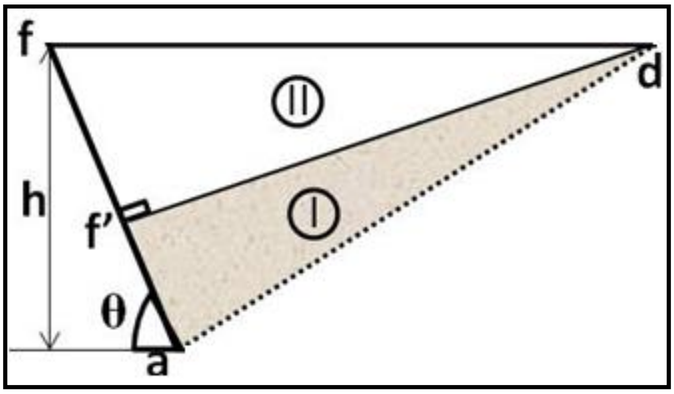
$$\Rightarrow \overline{\tau_b} = \frac{\rho g S h}{b}\left[b - \frac{h}{2}\left\{ Cosec\theta\left(\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)} \right) - 2\cot\theta \right\} \right] \qquad [4.68]$$

Average shear stress on wall $\overline{\tau_{wi}} = \rho g \frac{A}{P} S$

where, $A = A_I + A_{II} = \frac{1}{2} \times af \times df' = \frac{h\,\mathrm{cosec}\theta(z\sin\theta + h\cos\theta)}{2}$

and $P = af = h\,\mathrm{cosec}\theta$

Using equation[4.59], we get $\overline{\tau_{wi}} = \frac{\rho g S h}{2}\left[\frac{k_1}{(1-C)} \right]$ [4.69]



Fig[4.6]: Detail division of the left segment of fig [4.5]

Similarly, average shear stress on the right wall $\overline{\tau_{wo}} = \frac{\rho g S h}{2}\left[\frac{k_2}{(1+C)} \right]$ [4.70]

Using equations [4.68] and [4.69]

$$\frac{\overline{\tau_b}}{\overline{\tau_{wi}}} = \left(\frac{u_b^*}{u_{wi}^*} \right)^2 = k_1^{\,2} = \frac{\frac{\rho g S h}{b}\left[b - \frac{h}{2}\left(Cosec\theta\left(\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)} \right) - 2\cot\theta \right) \right]}{\rho g S h \frac{k_1}{2(1-C)}}$$

$$\frac{b}{h}\frac{k_1^{\,3}}{2(1-C)} + \frac{\mathrm{cosec}\theta}{2}\left[\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)} \right] - \cot\theta - \frac{b}{h} = 0 \qquad [4.71]$$

Similarly using eq.[4.68]and [4.70]

$$\frac{\overline{\tau_b}}{\tau_{wo}} = \left(\frac{u_b^*}{u_{wo}^*}\right)^2 = k_2{}^2 = \frac{\frac{\rho g S h}{b}\left[b - \frac{h}{2}\left(Co\sec\theta\left(\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)}\right) - 2\cot\theta\right)\right]}{\rho g S h \frac{k_2}{2(1+C)}}$$

$$\frac{b}{h}\frac{k_2{}^3}{2(1+C)} + \frac{\operatorname{cosec}\theta}{2}\left[\frac{k_1}{(1-C)} + \frac{k_2}{(1+C)}\right] - \cot\theta - \frac{b}{h} = 0 \qquad [4.72]$$

Solving eq.[4.71] and [4.72], we get,

$$\left[\frac{k_1}{k_2}\right] = \left[\frac{1-C}{1+C}\right]^{1/3}$$

This is similar to that of rectangular channel.

4.5.1.1 [Case II]: $\alpha = \alpha_{cr}$

At critical depth $\alpha = \alpha_{cr}$ and $h = h_{cr}$. Hence for point O in the flow field, the condition it should satisfy is,

$$\phi_{wi} = \phi_b = \phi_{wo}$$

Which, after substitution reduces to as,

$$\frac{b}{h_{cr}} = \operatorname{cosec}\theta\left[\frac{k_1}{1-C} + \frac{k_2}{1+C}\right] - 2\cot\theta \qquad [4.73]$$

Using equation [4.48] with [4.46] and [4.47] the following relation can be derived

$$k_1{}^3 + C - 1 = 0 \qquad [4.74]$$

$$k_2{}^3 - C - 1 = 0 \qquad [4.75]$$

Equation [4.74] and [4.75] are similar to that of the rectangle.

Now, using all above three eq[4.73],[4.74]and[4.75]

$$\frac{b}{h_{cr}} - \operatorname{cosec}\theta\left[\frac{1}{(1-C)^{2/3}} + \frac{1}{(1+C)^{2/3}}\right] + 2\cot\theta = 0 \qquad [4.76]$$

The eq[4.76] can be solved to find the critical depth for a trapezoidal channel, which is a function of the channel geometry.

4.5.1.2 [Case III]: $\alpha \leq \alpha_{cr}$

Here the water level lies above the critical depth. So for any possible position the division could be carried out as shown in fig[4.7], acfg is the flow area in which ao and oc are the inclined division lines found by using the equation explained in the previous case up to the critical depth. While, division line ot is found by equating the relative distance of each point on it to left and right wall,

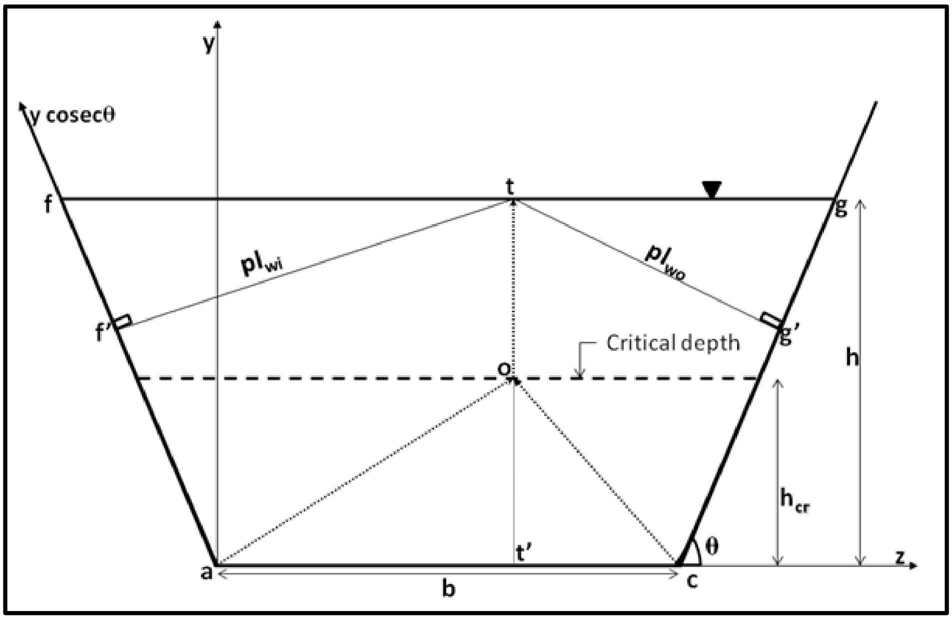
$\phi_{wi} = \phi_{wo}$, substituting the necessary parameter the equation becomes

$$\frac{pl_{wi}}{pl_{wo}} = \frac{k_1[1+C]}{k_2[1-C]}$$

From fig.[4.7]

$$\frac{z\sin\theta + h\cos\theta}{(b-z)\sin\theta + h\cos\theta} = \frac{k_1[1+C]}{k_2[1-C]}$$

$$\Rightarrow z = \frac{k_1 b[1+C]}{k_1[1+C] + k_2[1-C]} + h\cot\theta \qquad [4.77]$$



Fig[4.7]: For $\alpha \leq \alpha_{cr}$

Area contributing energy to the inner wall, A_{wi} =area (aotf) = area(aftt')-area(aot')

or, $A_{wi} = \frac{1}{2}(2z + h\cot\theta)h - \frac{1}{2}zh_{cr} = \frac{1}{2}\left[z(2h - h_{cr}) + h^2\cot\theta\right]$

Similarly, area corresponding to bed, A_b and outer wall, A_{wo} is,

$$A_b = \frac{1}{2}bh_{cr}$$

$$A_{wo} = bh - A_{wi} - A_{wb} = \frac{1}{2}\left[(b - z)(2h - h_{cr}) + h^2\cot\theta\right]$$

So, average shear stress can be calculated by-

$$\bar{\tau}_{wi} = \rho g \frac{A_{wi}}{h\operatorname{cosec}\theta} S, \qquad \bar{\tau}_b = \rho g \frac{A_b}{b} S \qquad \text{and} \qquad \bar{\tau}_{wo} = \rho g \frac{A_{wo}}{h\operatorname{cosec}\theta} S \quad [4.78]$$

4.6 Helical flow effect

Due to substantial curvature in the meandering channel the secondary flow and the primary combines to form helical flow, which deviates the streamline near the surface toward outer bank and the streamlines near bed get deviated towards the inner bank. This helicoidal flow gradually develops as it proceeds toward the bend apex of a channel and then the intensity starts decreasing as it leaves the section. This complex flow mechanism distributes certain fraction of energy over the boundary of the channel. Due to this helix, a fraction of energy

Crosato [2008], the effect of varying effect of helical flow is an indirect way of studying the effect of curvature over the flow. Curvature ratio i.e the ratio between the channel width and the radius of curvature, and the aspect ratio of channel are important parameter to quantify the secondary flow induced by curvature into the channel, Blanckaert & de Vriend [2003]. Hence there must be certain relation between these parameters viz. curvature ratio and channel aspect ratio with the distribution.

The above formulation is solely based theoretical analysis and assumption. Since, fluid problems cannot be easily analyzed only by theory; and the theories are often essentially supplemented by the experimental findings.

Taking the experimental data, a genetic programming based software is used to find an empirical relationship within the above mentioned parameters. Two different relationships, one for rectangular and other for trapezoidal channel is found. This are forces due to helical flow acting on the walls in addition to the forces calculated in the above section for rectangular or trapezium meandering channel.

For Rectangular Channel:

$$HSF_{rec}\% = \left(\frac{\log((\exp(c_o.b/h) + c_1}{(c_3.b/r + c_4)} + c_5 \right) \qquad [4.79]$$

where, c_0=3.09, c_1=-14.75, c_2=-5.84, c_3=15.82, c_4=-12.42, c_5=25.46

For Trapezoidal channel:

$$HSF_{trap}\% = 0.064 - 1.027\exp\left(\frac{0.021}{-8.716 b/r - 0.01 b/h + 2.31} \right) \qquad [4.80]$$

In both of the equation b/h represents aspect ratio and b/r represents circumference ratio.

Using the above equations, the average percentage shear force acting on the wall will be,

$$SF_{\mathrm{Rect}}\% = \frac{(\bar{\tau}_{wi} * h)}{(\bar{\tau}_{wi} + \bar{\tau}_{wo})h + b\bar{\tau}_b} \pm \left(\frac{\log((\exp(c_o.b/h) + c_1}{(c_3.b/r + c_4)} + c_5 \right)$$

$$SF_{trap}\% = \frac{(\bar{\tau}_{wi} * h\,\mathrm{cosec}\theta)}{(\bar{\tau}_{wi} + \bar{\tau}_{wo})h\,\mathrm{cosec}\theta + b\bar{\tau}_b} \pm \left(0.064 - 1.027\exp\left(\frac{0.021}{-8.716 b/r - 0.01 b/h + 2.31} \right) \right)$$

Here, positive sign is for inner wall and negative for outer wall respectively.

CHAPTER V

MODEL VALIDATION

5.1 Overview

In the Chapter V, a model with equations were developed for computing average shear stress in the bed, inner and outer wall of a simple meandering channel. The model will be validated in this section with the experimental data.

5.2 Sources of Data

The data used for analysis are-

For Rectangular Channel:

1. Experimental findings from the meandering channel of sinuosity 1.22 by S. K. Kar (1977) at IIT, Kharagpur.

2. A.K. Das (1984) carried out similar experimentation on the meandering channel at IIT Kharagpur is used here for the analysis

3. Data sets from similar experimentations conducted at NIT, Rourkela by K.K. Khatua (2008) having sinuosity 1.44.

For Trapezoidal Channel:

4. Experimental data of a meandering channel with sinuosity 1.91 observed by K.K.Khatua(2008) at NIT, Rourkela.

5. Experimentation done by M. Pattnaik(2013) on trapezoidal meandering channel of sinuosity 2.04 at NIT, Rourkela.

6. Data of another meandering channel with sinuosity 4.11 whose experimentation was done by Arpan Pradhan(2014) at NIT, Rourkela.

The detailed parameters of all the datasets are given in table 5.1.

Table [5.1]: Channel Characteristics of different Meandering Channel

Test Series	Data Set	No. of Runs	Channel bottom width, b (in m)	Mean Radius, r (in m)	Side Slope, θ (in degree)	Sinuosity, Sr	Valley Slope, Sv	Manning's n	Remark
1	Kar (1977)	3	0.1	0.06	90	1.22	0.0061	0.009	Smooth
2	Das (1984)	5	0.1	0.11	90	1.22	0.004	0.009	Smooth
3	Khatua (2008)	4	1.2	0.14	90	1.44	0.0031	0.011	Smooth
4	Khatua (2008)	6	0.12	0.46	45	1.91	0.0053	0.011	Smooth
5	Patnaik (2013)	6	0.33	1.01	45	2.04	0.0055	0.01	Smooth
6	Pradhan (2014)	3	0.33	1.43	45	4.11	0.00165	0.01	Smooth

5.3 Critical Depth Computation

As per the model, critical depth for each channel is computed using equation [4.40] and [4.42] for rectangular section and [4.76] for trapezoidal section. A sample solution of this equation is showed in Appendix-I. The critical depth for all channel used for validation is tabulated in Table-5.2

Table [5.2]: Computed Critical Depth

Test Series	Channel width(in m)	Critical depth (in m)	Critical Aspect Ratio, α_{cr}
1	1.0	0.377	2.65
2	1.0	0.480	2.08
3	1.2	0.589	2.04
4	0.12	0.137	0.878
5	0.33	0.346	0.953
6	0.33	0.398	0.830

5.4 Results

The following next figure shows the percentage shear force distribution for the derived model in comparison to experimental distribution for each run for each series:

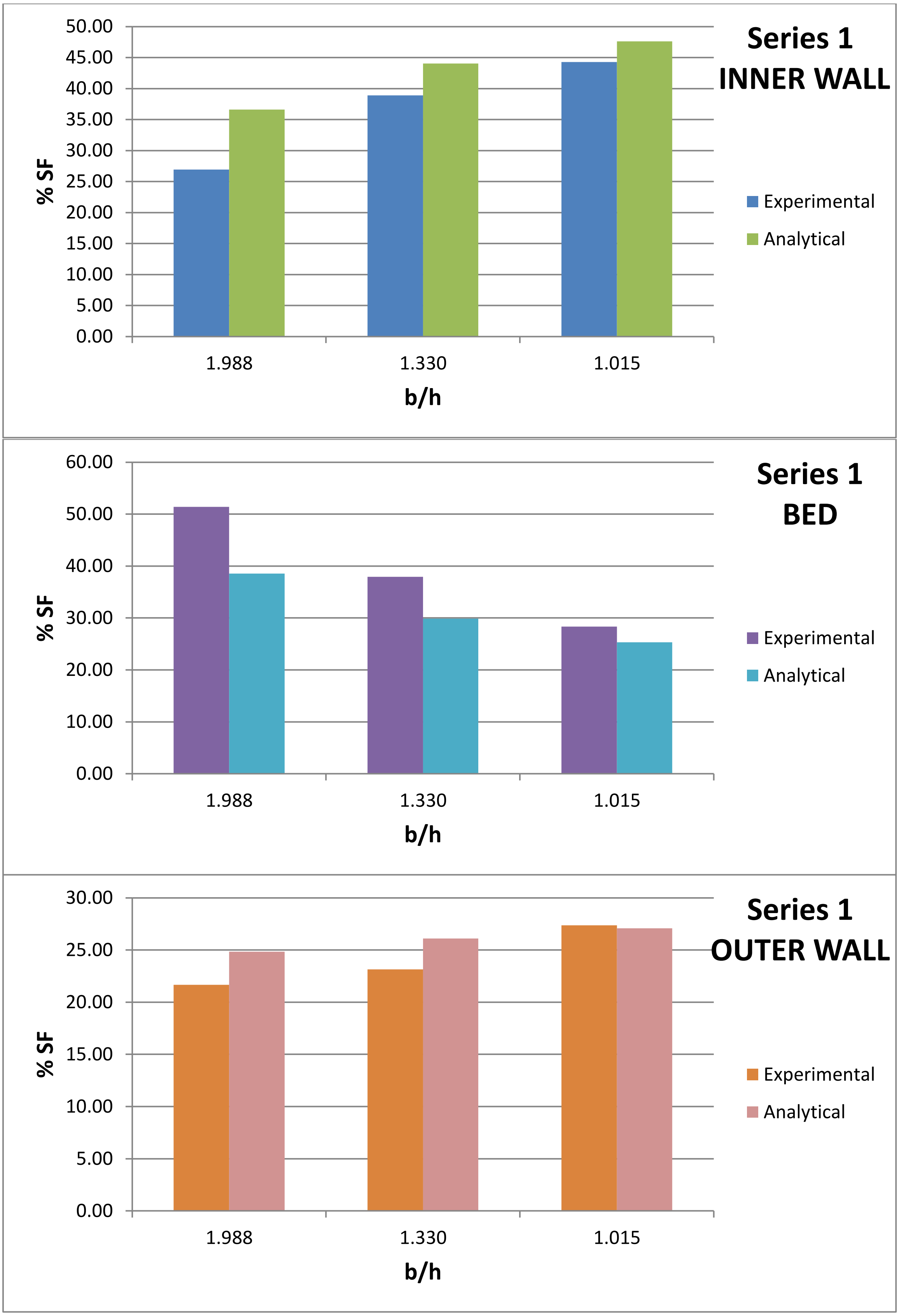


Fig. [5.1]: Percentage shear force at the inner wall, bed and outer wall of meandering channel of sinuosity 1.22 by S. K. Kar (1977)

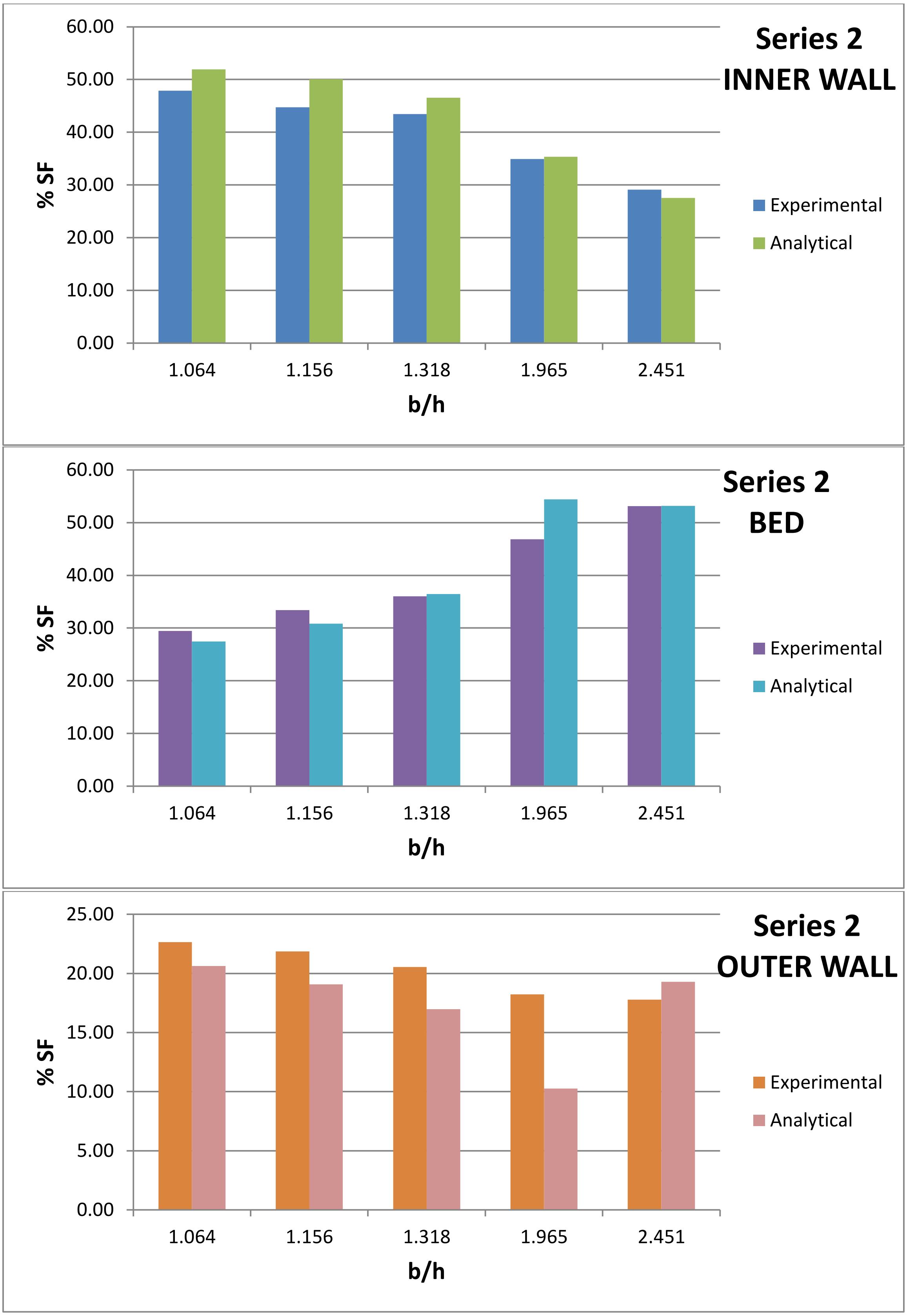


Fig. [5.2]: Percentage shear force at the inner wall, bed and outer wall of meandering channel of sinuosity 1.22 by A.K. Das (1984)

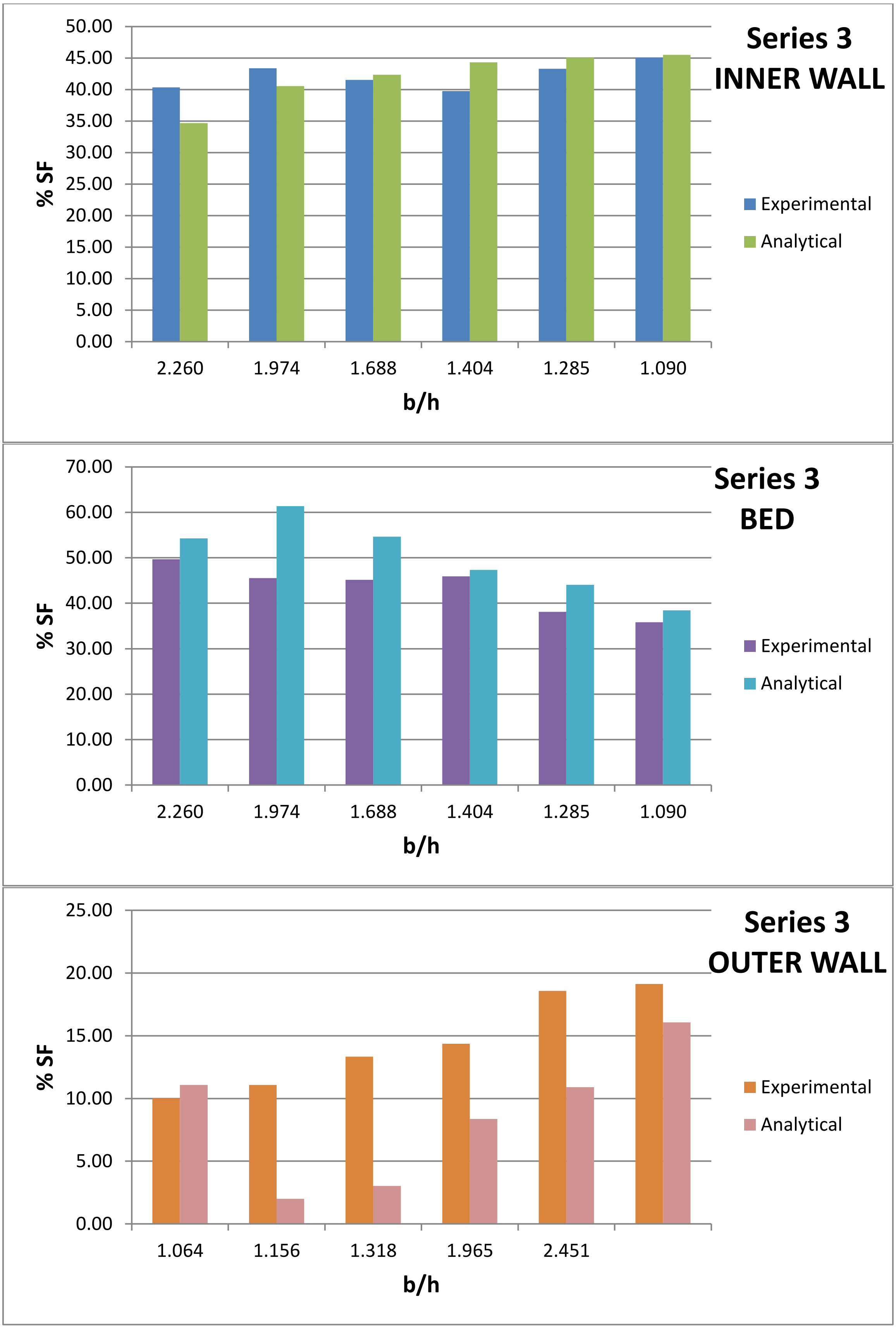


Fig. [5.3]: Percentage shear force at the inner wall, bed and outer wall of meandering channel of sinuosity 1.44 by K.K. Khatua (2008)

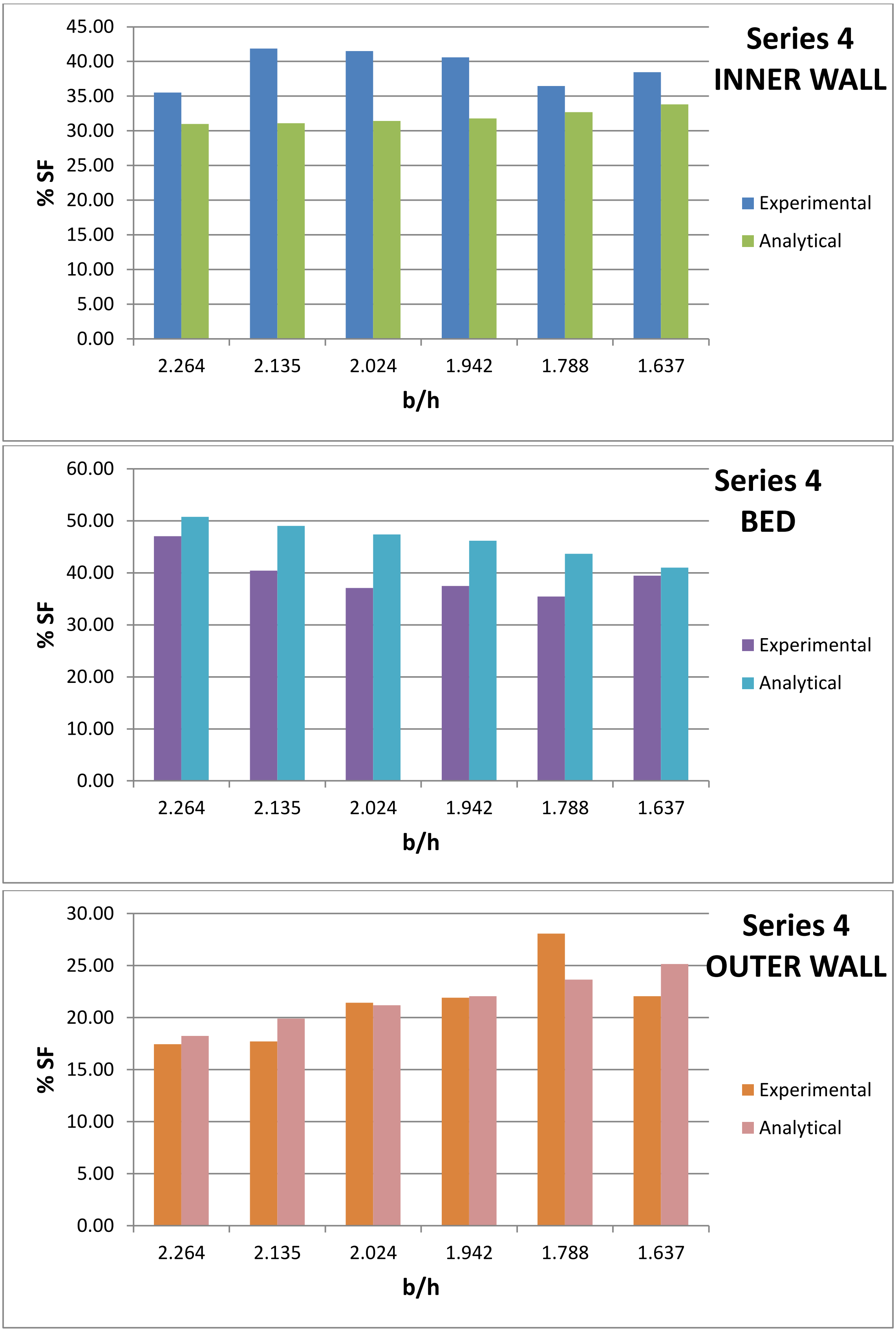


Fig. [5.4]: Percentage shear force at the inner wall, bed and outer wall of trapezoidal meandering channel of sinuosity 1.91 by K.K.Khatua(2008)

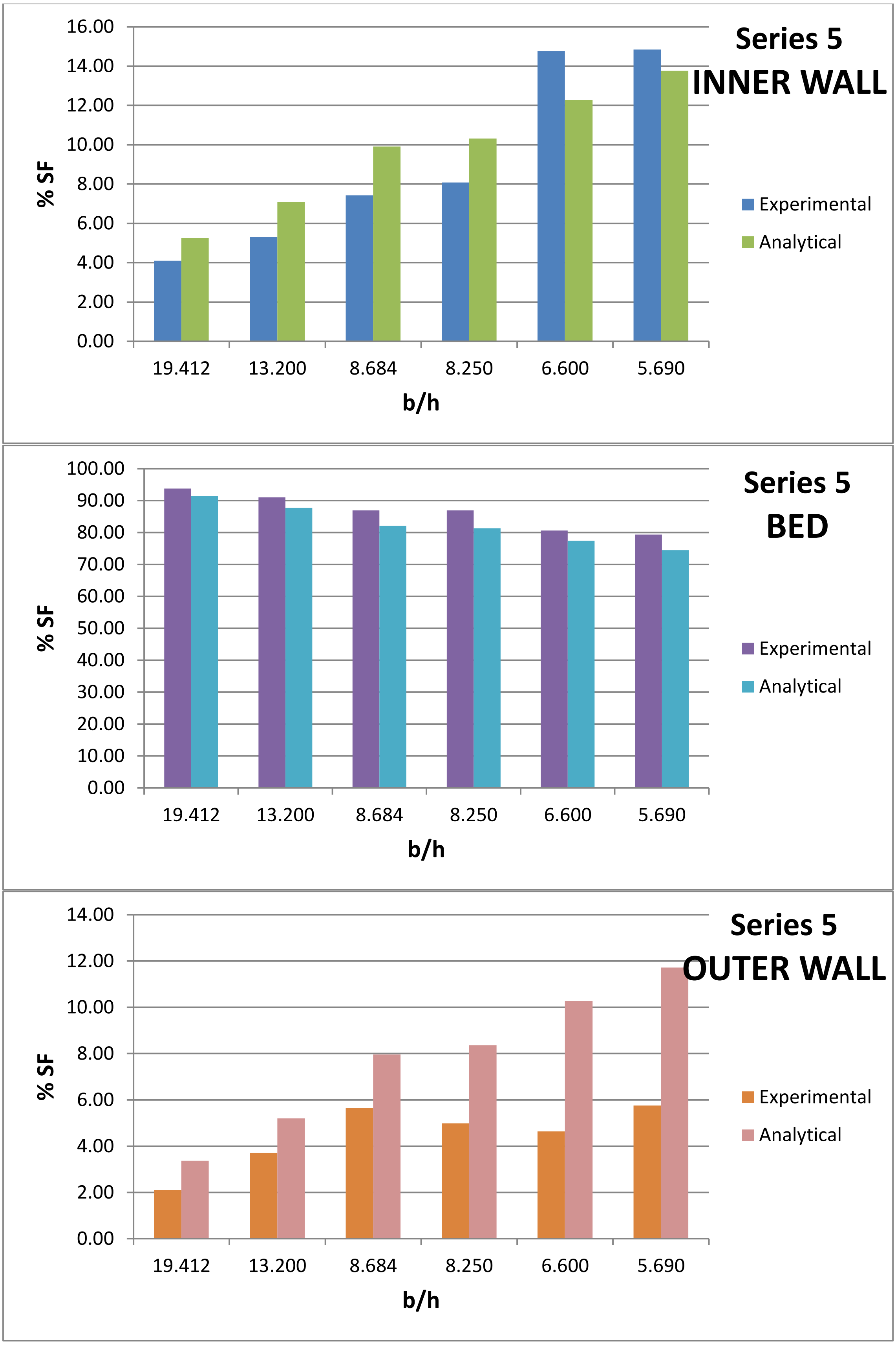


Fig. [5.5]: Percentage shear force at the inner wall, bed and outer wall of trapezoidal meandering channel of sinuosity 2.04 by M. Pattnaik(2013)

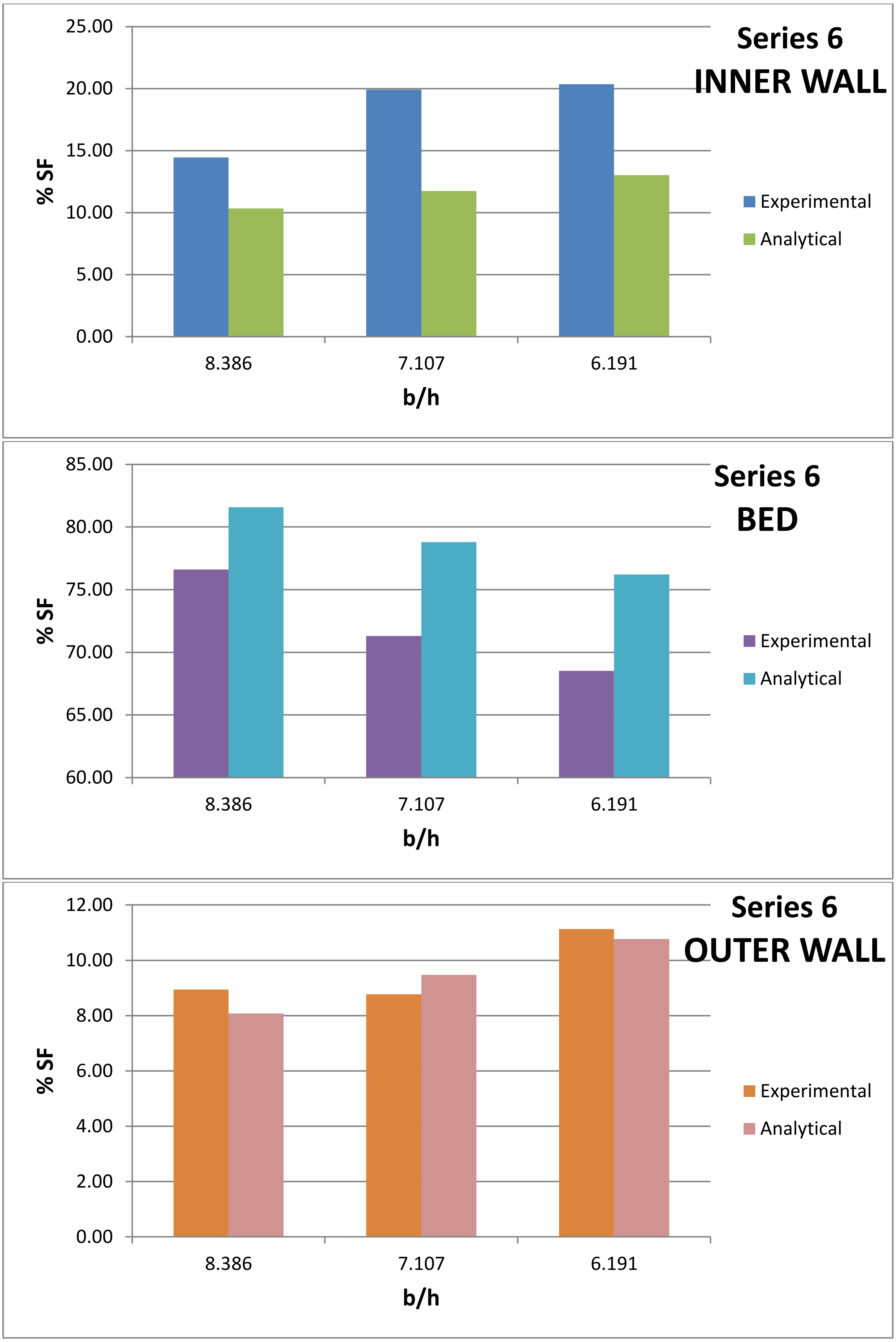


Fig. [5.6]: Percentage shear force at the inner wall, bed and outer wall of trapezoidal meandering channel of sinuosity 4.11 by Arpan Pradhan(2014)

Table [5.3]: Tabulation of Calculated Values for k1, k2 and Shear Stress Distribution for different aspect ratios of the datasets

Test Series	Curvature Ratio, b/r	Aspect Ratio, b/h	Centrifugal Ratio, C	k1	k2	Percentage Distribution of Shear Force in inner wall (in), bed and outer wall (out)						Percentage Error		
						Analytical			Experimental					
						in	bed	out	in	bed	out	in	bed	out
1	1.667	1.988	1.047	-0.164	0.578	36.61	38.55	24.84	26.92	51.41	21.68	9.69	-12.86	3.17
	1.667	1.330	1.429	-0.623	1.111	44.03	29.87	26.10	38.91	37.94	23.15	5.12	-8.07	2.95
	1.667	1.015	2.011	-0.895	1.287	47.59	25.34	27.07	44.27	28.36	27.38	3.33	-3.02	-0.30
2	0.909	1.064	0.895	0.189	0.495	51.91	27.46	20.63	47.89	29.46	22.64	4.01	-2.00	-2.01
	0.909	1.156	0.826	0.306	0.669	50.07	30.85	19.08	44.72	33.41	21.87	5.35	-2.57	-2.79
	0.909	1.318	0.719	0.467	0.854	46.55	36.48	16.97	43.44	36.02	20.54	3.12	0.45	-3.57
	0.909	1.965	0.567	0.707	1.086	35.35	54.40	10.26	34.94	46.83	18.23	0.40	7.57	-7.97
	0.909	2.451	0.435	0.844	1.151	27.54	53.16	19.30	29.11	53.12	17.77	-1.57	0.05	1.52
3	0.857	2.260	0.100	0.992	1.061	34.66	54.27	11.07	40.33	49.67	10.00	-5.67	4.60	1.07
	0.857	1.974	0.104	0.959	1.029	40.54	61.37	2.00	43.37	45.55	11.08	-2.83	15.82	-9.08
	0.857	1.688	0.111	0.916	0.987	42.33	54.64	3.03	41.53	45.14	13.33	0.80	9.50	-10.30
	0.857	1.404	0.121	0.856	0.929	44.32	47.31	8.37	39.76	45.89	14.35	4.56	1.43	-5.99
	0.857	1.285	0.126	0.826	0.898	45.05	44.06	10.89	43.31	38.12	18.57	1.74	5.94	-7.68
	0.857	1.090	0.135	0.764	0.836	45.50	38.44	16.06	45.06	35.82	19.13	0.45	2.62	-3.07

Table [5.4]: Tabulation of Calculated Values for k1, k2 and Shear Stress Distribution for different aspect ratios of the datasets

Test Series	Curvature Ratio, b/r	Aspect Ratio, b/h	Centrifugal Ratio, C	k1	k2	Percentage Distribution of Shear Force in inner wall (in), bed and outer wall (out)						Percentage Error		
						Analytical			Experimental					
						in	bed	out	in	bed	out	in	bed	out
4	0.261	2.264	0.046	1.118	1.153	31.01	50.75	18.24	35.51	47.03	17.46	-4.50	3.72	0.78
	0.261	2.135	0.049	1.110	1.147	31.08	49.00	19.91	41.87	40.41	17.72	-10.79	8.59	2.19
	0.261	2.024	0.051	1.103	1.142	31.41	47.40	21.19	41.49	37.07	21.43	-10.08	10.32	-0.24
	0.261	1.942	0.053	1.098	1.138	31.78	46.15	22.07	40.59	37.49	21.91	-8.82	8.66	0.16
	0.261	1.788	0.058	1.087	1.129	32.68	43.67	23.65	36.47	35.47	28.07	-3.79	8.21	-4.42
	0.261	1.637	0.063	1.074	1.12	33.82	41.02	25.16	38.47	39.46	22.06	-4.65	1.56	3.09
5	0.326	19.412	0.004	1.242	1.245	5.25	91.39	3.36	4.11	93.79	2.10	1.14	-2.40	1.26
	0.326	13.200	0.004	1.234	1.238	7.10	87.69	5.20	5.31	90.98	3.71	1.79	-3.29	1.50
	0.326	8.684	0.007	1.221	1.227	9.91	82.13	7.96	7.43	86.93	5.64	2.48	-4.80	2.32
	0.326	8.250	0.008	1.218	1.225	10.32	81.32	8.36	8.08	86.94	4.99	2.24	-5.62	3.38
	0.326	6.600	0.010	1.209	1.217	12.29	77.43	10.28	14.76	80.60	4.64	-2.47	-3.17	5.64
	0.326	5.690	0.011	1.201	1.21	13.77	74.51	11.72	14.84	79.40	5.75	-1.08	-4.89	5.97
6	0.231	8.386	0.009	1.219	1.226	10.33	81.58	8.08	14.45	76.61	8.94	-4.11	4.97	-0.86
	0.231	7.107	0.009	1.212	1.22	11.73	78.79	9.48	19.92	71.31	8.78	-8.18	7.48	0.70
	0.231	6.191	0.008	1.206	1.213	13.02	76.21	10.77	20.35	68.52	11.13	-7.34	7.69	-0.35

Predicted values of k1 and k2 for different series:-

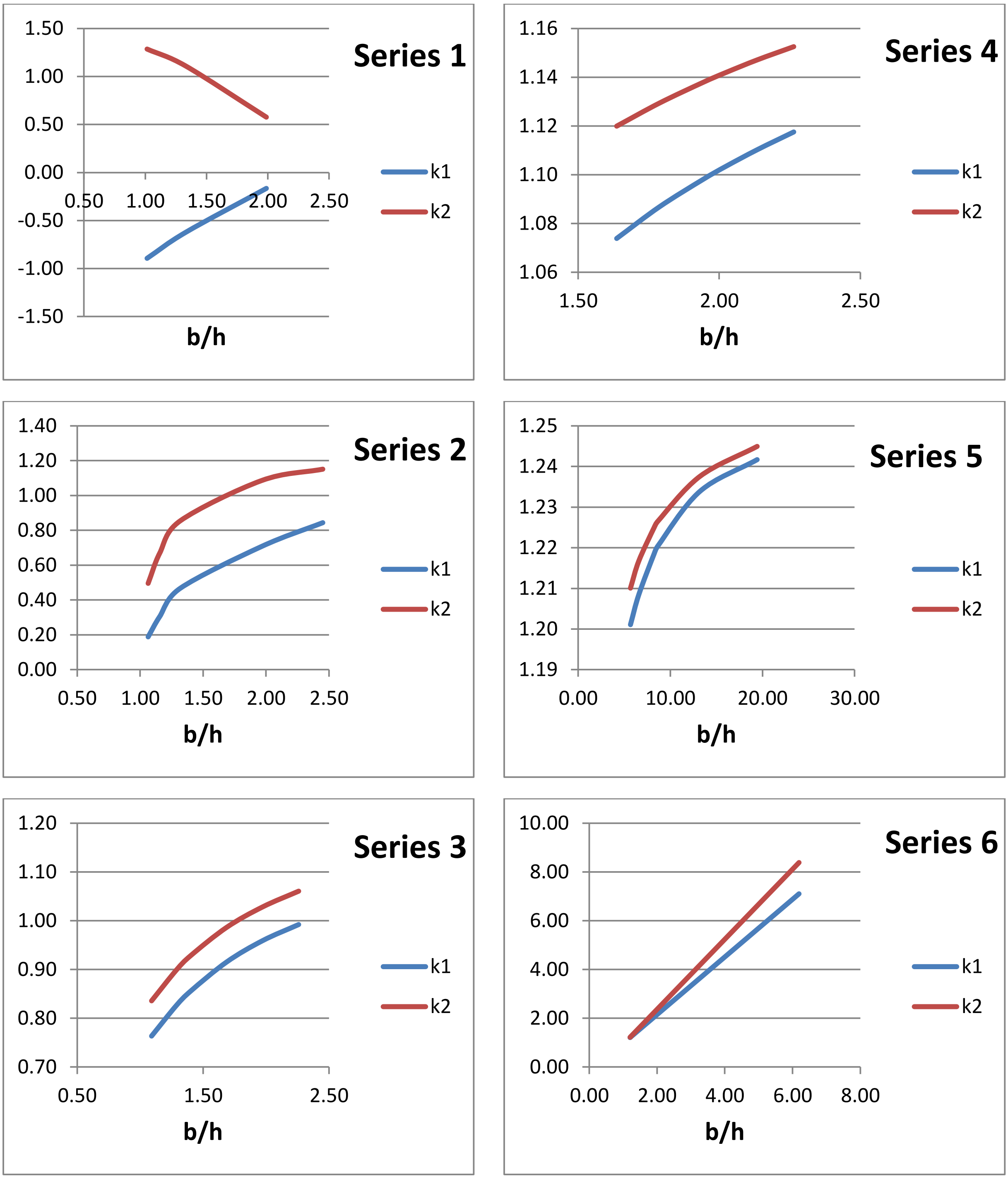


Fig [5.7]: k1 and k2 variation with respect to the aspect ratio (b/h) for different channels

From the above figure it can be observed that for series 1 the general trend of k1 and k2 variation is different from others. This is because for series 1 the value of centrifugal ratio is higher than 1 which gives rise to negative ratio between k1 and k2.

CHAPTER VI

SUMMARY AND CONCLUSION

Summary and Conclusion

An analytical approach for the computation of BSS distribution along the wetted perimeter of a meandering channel has been developed. The approach partitions the cross-sectional area to determine the differential contribution of flow towards the shear distribution along the channel bed and the inner and outer side walls.

The approach is based on the mechanism and direction of energy transportation for turbulent flow in a meandering channel, by the order-of-magnitude method based on the Reynolds equations in a steady and uniform flow channel. It is reasoned that the surplus energy contained in any flow volume, for unit length in the flow direction, will be transferred toward a unit area on the wetted boundary. The direction of energy transportation is along the shortest geometrical distance, ψ, between the source concerned and the boundary.

The method is illustrated in a study of the boundary shear stress distribution in smooth rectangular open channels. Analytical solutions, valid for all aspect ratios, are derived for the mean side wall and bed shear stresses, and they compared well with experimental data from various researchers. Comparisons of the computed and measured mean shear stress distributions along the side wall and bed are acceptable.

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Critical Depth Determination

To determine the critical depth 'Mathematica', mathematical computing software is used to solve the equation, which is a non-linear implicit function, to find the critical depth and then the critical aspect ratio. The values that were computed for each channel is given in table 5.2 and a sample program is shown below.

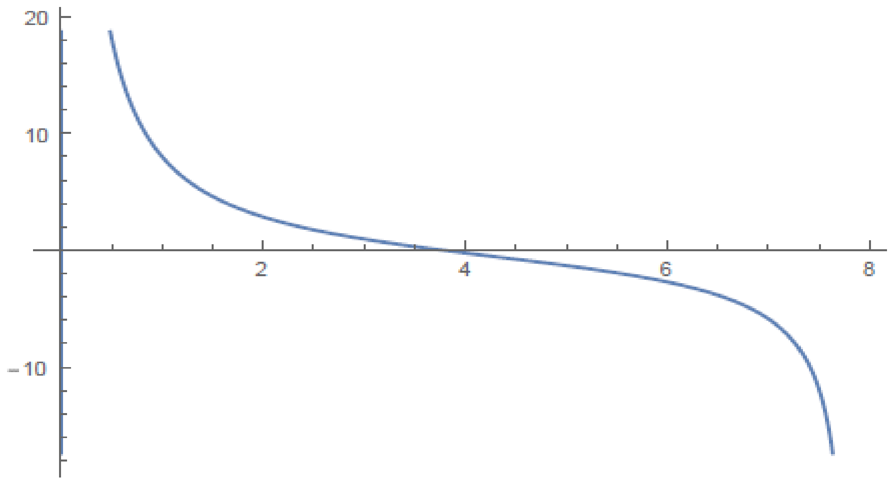
```
In[1]:= b = 10; S = 0.005; R = 6; g = 981; n = 0.009 * 100 ^ ( -1 / 3) ;
        a = S / ((n ^ 2) g R) ;
        c = a * (((b h) / (b + 2 h)) ^ (4 / 3)) ;
        equ = (b / h) - ((1 - c) ^ ( -2 / 3)) - ((1 + c) ^ ( -2 / 3))
```

Out[4]=

$$\frac{10}{h} - \frac{1}{\left(1 - 4.86779 \left(\frac{h}{10 + 2\,h}\right)^{4/3}\right)^{2/3}} - \frac{1}{\left(1 + 4.86779 \left(\frac{h}{10 + 2\,h}\right)^{4/3}\right)^{2/3}}$$

```
In[5]:= Plot[equ, {h, -0.1, 8}]
```

Out[5]=




```
In[6]:= FindInstance[equ == 0, h, Reals]
Out[6]= { {h → 3.77055} }
```

Fig[4.1] Sample Program to Calculate Critical Depth in 'Mathematica'

MATLAB Programming

To determine the shear stresses acting on the wall and side walls the following program can be used. The program with a sample run is shown below. The code is in the box and the output of each section run is shown just below the box.

Program:-

```
%Program to calculate the k1 and k2 value as well as the shear
stress on the wall and bed.
```

```
%% REQUIRED DATA
disp('Enter all values in SI unit')
r=input('Mean Radius :');
b=input('Bottom Width :');
a=input('Side Slope Angle in Degrees :');
n=input('Mannings Coefficient :');
hcr=input('Critical Depth of the Channel :');
h= input('Water Depth :');
s=input('Energy Slope :');
```

Enter all values in SI unit

Mean Radius :0.46

Bottom Width :0.12

Side Slope Angle in Degrees :45

Mannings Coefficient :0.01

Critical Depth of the Channel :0.137

Water Depth :0.053

Energy Slope :0.001899

```
%% CALCULATIONS
AR=b/h;%aspect ratio
A=(b+(h*cotd(a)))*h;%wetted area
P=b+(2*h*cscd(a));%wetted perimeter
R=A/P;%hydraulic radius
k=s/((n^2)*9.81*r);
C=k*(R^(4/3));
rat=(1-C)/(1+C);
if rat<0
    ratio=(-1)*((-1*rat)^(1/3));
else
    ratio=rat^(1/3);
end
```

```
%% FOR k1 AND k2
a1=AR*(0.5/(1+C));
b1=0;
c1=0.5*cscd(a)*((1/(1+C))+(ratio/(1-C)));
d1=-1*(cotd(a)+AR);
p=[a1 b1 c1 d1];
k22=roots(p);
for i=1:3
    if imag(k22(i,1))==0
        k2=k22(i,1)
    end
end
k1=k2*ratio
```

k2 = 1.1526

k1 = 1.1175

```
%% SHEAR STRESS CALCULATION
if h <= hcr
    %SHEAR STRESS FOR LOWER DEPTH
    bed=((1000*9.81*s*h)/b)*(b-((h/2)*((cscd(a)*((k1/
         (1-C))+(k2/(1+C))))-(2*cotd(a)))));
    inner_wall=(1000*9.81*s*h*k1)/(2*(1-C));
    outer_wall=(1000*9.81*s*h*k2)/(2*(1+C));
else
    %SHEAR STRESS FOR HIGHER DEPTH
    Z=(b*k1*(1+C))/((k1*(1+C))+(k2*(1-C)))+(h*cotd(a));
    Awi=(h*Z)-(0.5*Z*hcr)+(0.5*(h^2)*cotd(a));
    Ab=0.5*b*hcr;
    Awo=(b*h)-Awi-Ab+((h^2)*cotd(a));
    bed=1000*9.81*s*Ab/b;
    inner_wall=1000*9.81*s*Awi/(h*cscd(a));
    outer_wall=1000*9.81*s*Awo/(h*cscd(a));
end
```

```
%% DISPLAY
disp('shear stress in:- ')
disp('bed :')
disp(double(bed))
disp('inner wall :')
disp(double(inner_wall))
disp('outer wall :')
disp(double(outer_wall))
```

shear stress in:-

bed : 0.7224

inner wall : 0.5785

outer wall : 0.5438